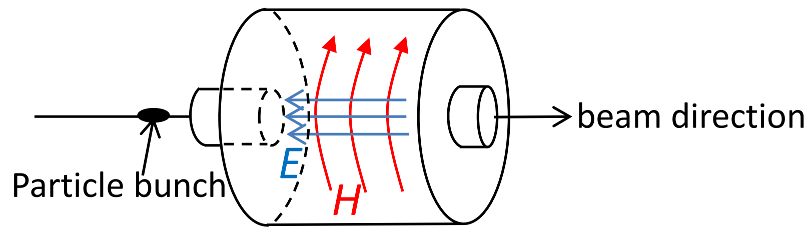
Basic principles of RF Superconductivity 1/2

Tobias Junginger

Particle Acceleration with cavities

1. Acceleration with radiofrequency cavities requires synchronization of particles and RF field. Outside of the cavity the approaching particle bunch does not experience the RF field



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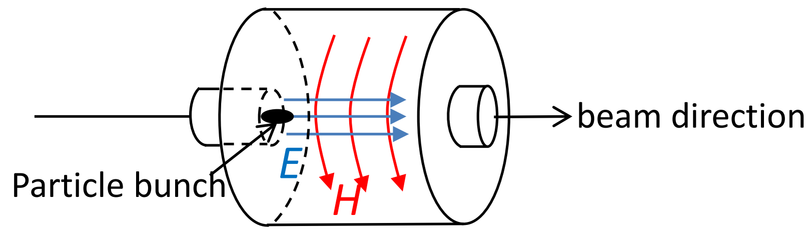

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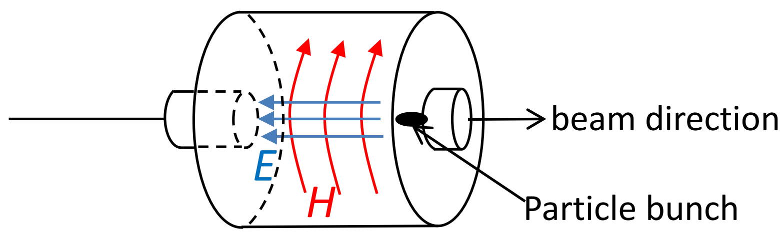

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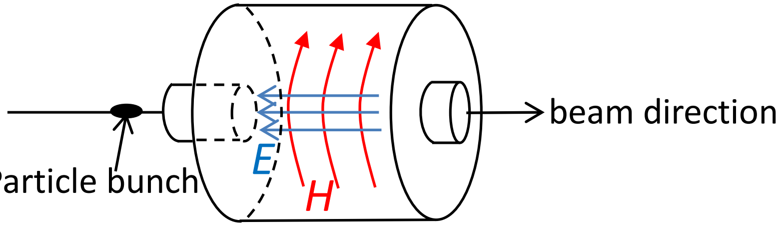

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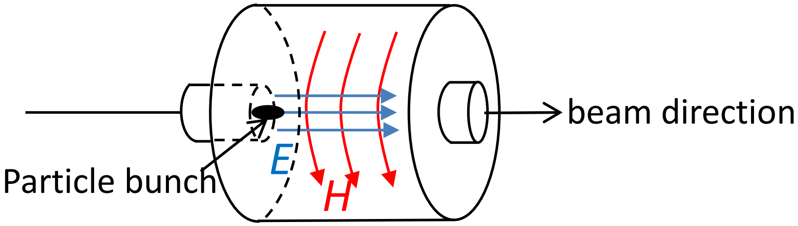

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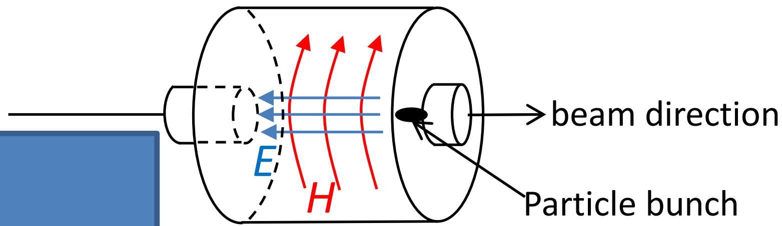

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- An oscillating electric field causes an oscillating magnetic field
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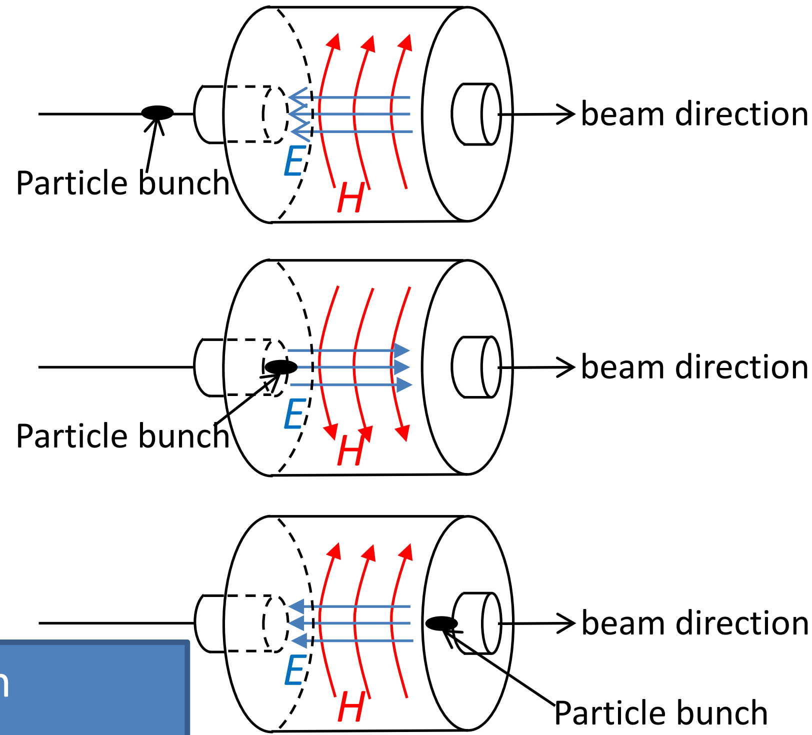

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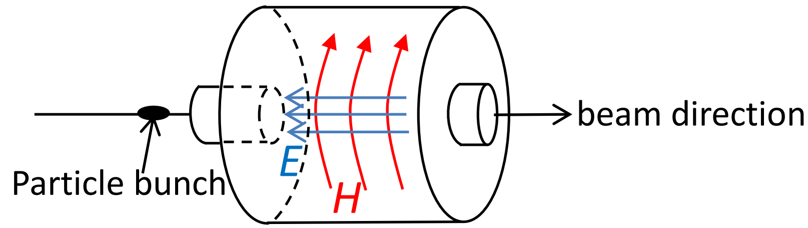

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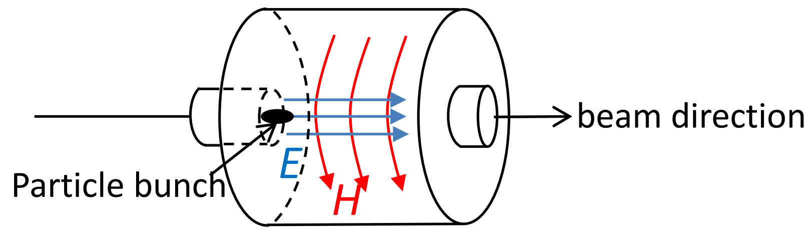

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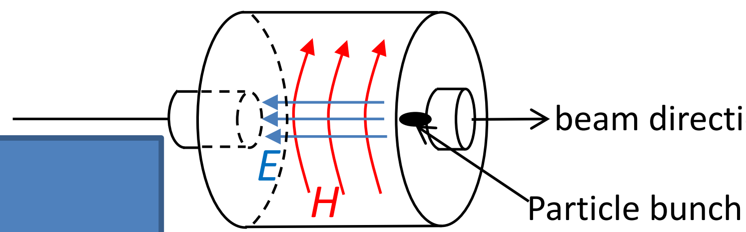

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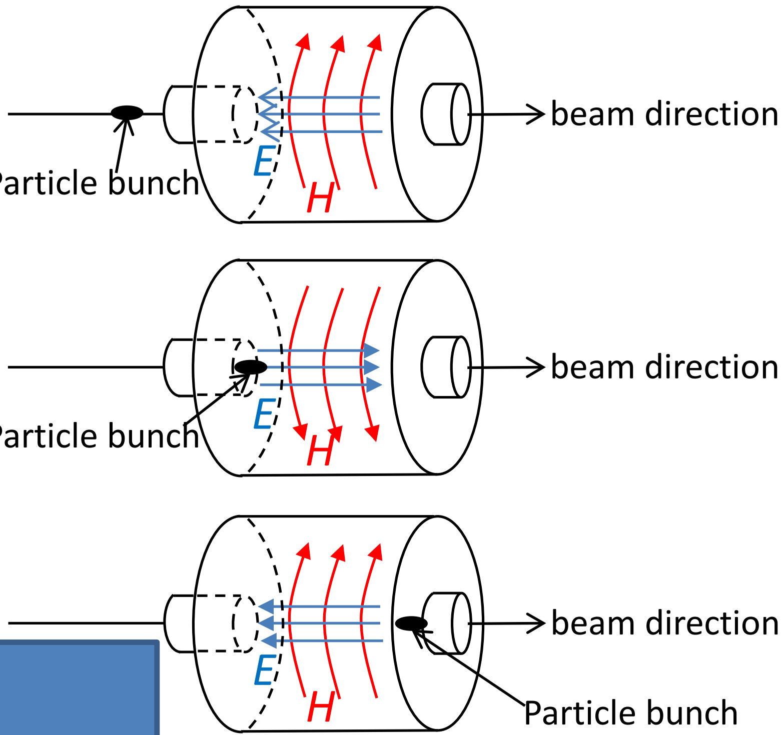

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Particle Acceleration with cavities

In this lecture we will address these questions:

- Superconductivity means no resistance. Why can't we reduce the losses zero?
- Why is niobium the material choice which requires costly helium cooling?
- What are the fundamental and technical limitations of niobium SRF cavities?
  - Highest Energy Gain → Maximum Accelerating Gradient?
  - Lowest cryogenic losses → Maximum Quality Factor?
- What are possible future materials and what are the challenges?

→beam direction

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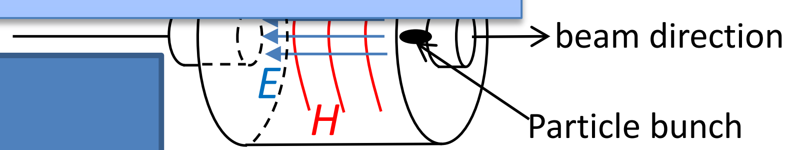
field direction has changed again



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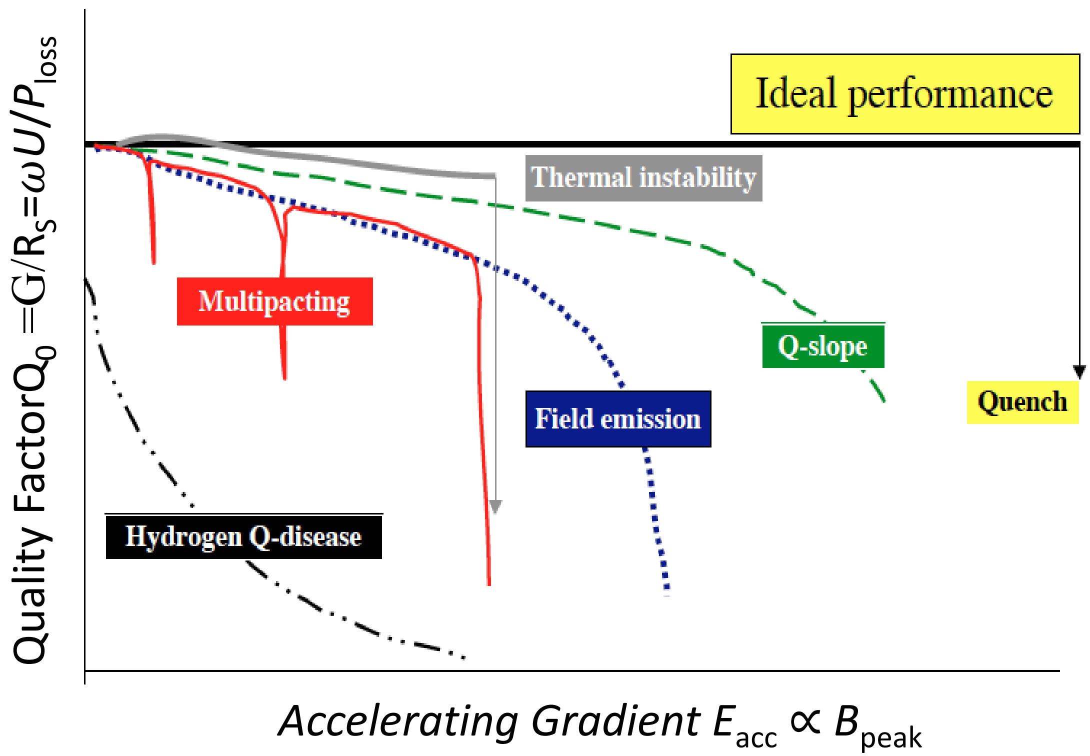
Performance of SRF cavities

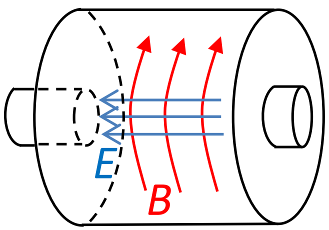
There are two parameters which define the performance of an SRF cavity: **Quality factor and the accelerating gradient**



The quality factor:

Q_0 =G/R_S=$\omega U/P_{loss}$
G: Geometry factor

The accelerating gradient can be limited by the peak surface electric field (field emission) or the peak surface magnetic field (quench)

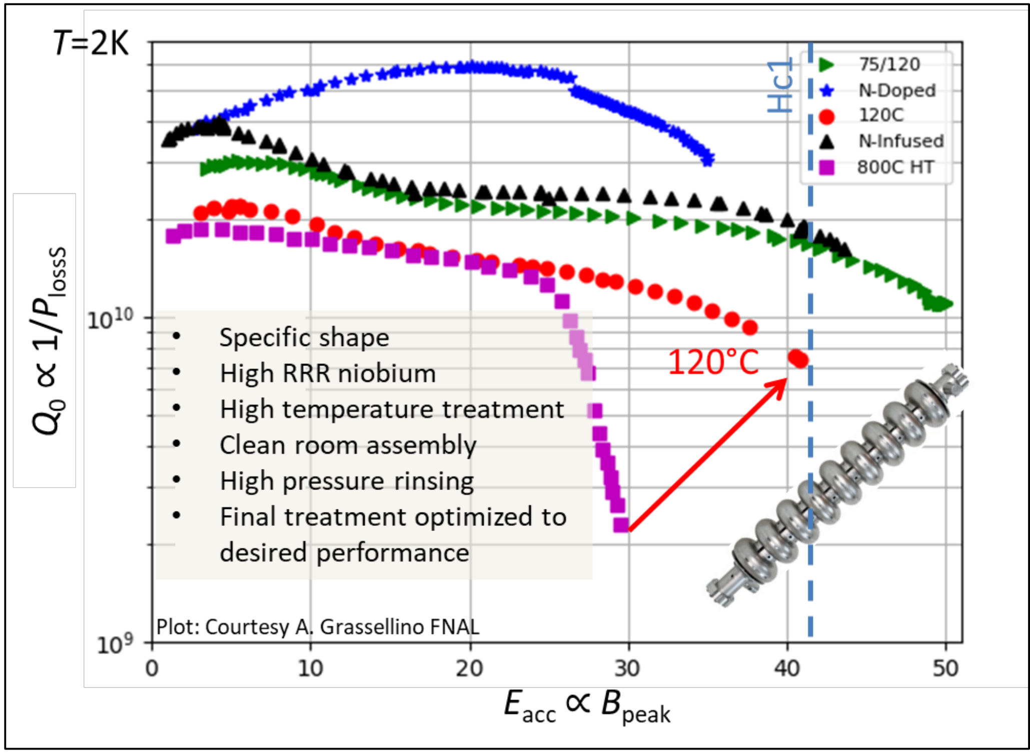


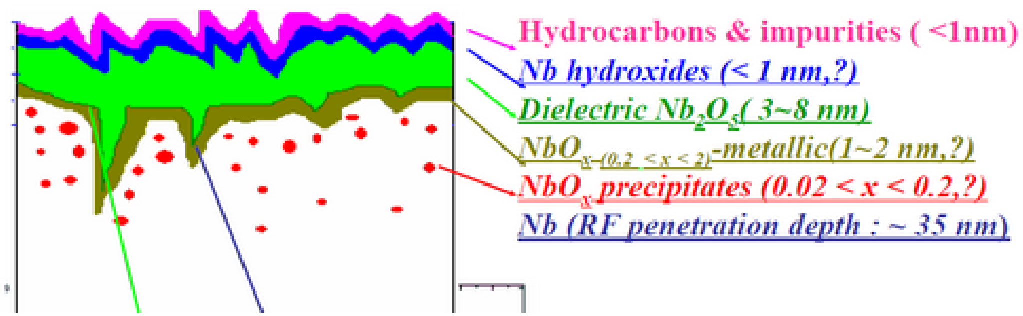
There are two ways to increase performance: Shape and material optimization
In this lecture the focus is on material optimization. **What are the intrinsic limitations to R_s and B_{peak}**

Shape optimization and extrinsic limitations, i.e Multipacting, field emission and thermal breakdown are covered in Bob's lecture

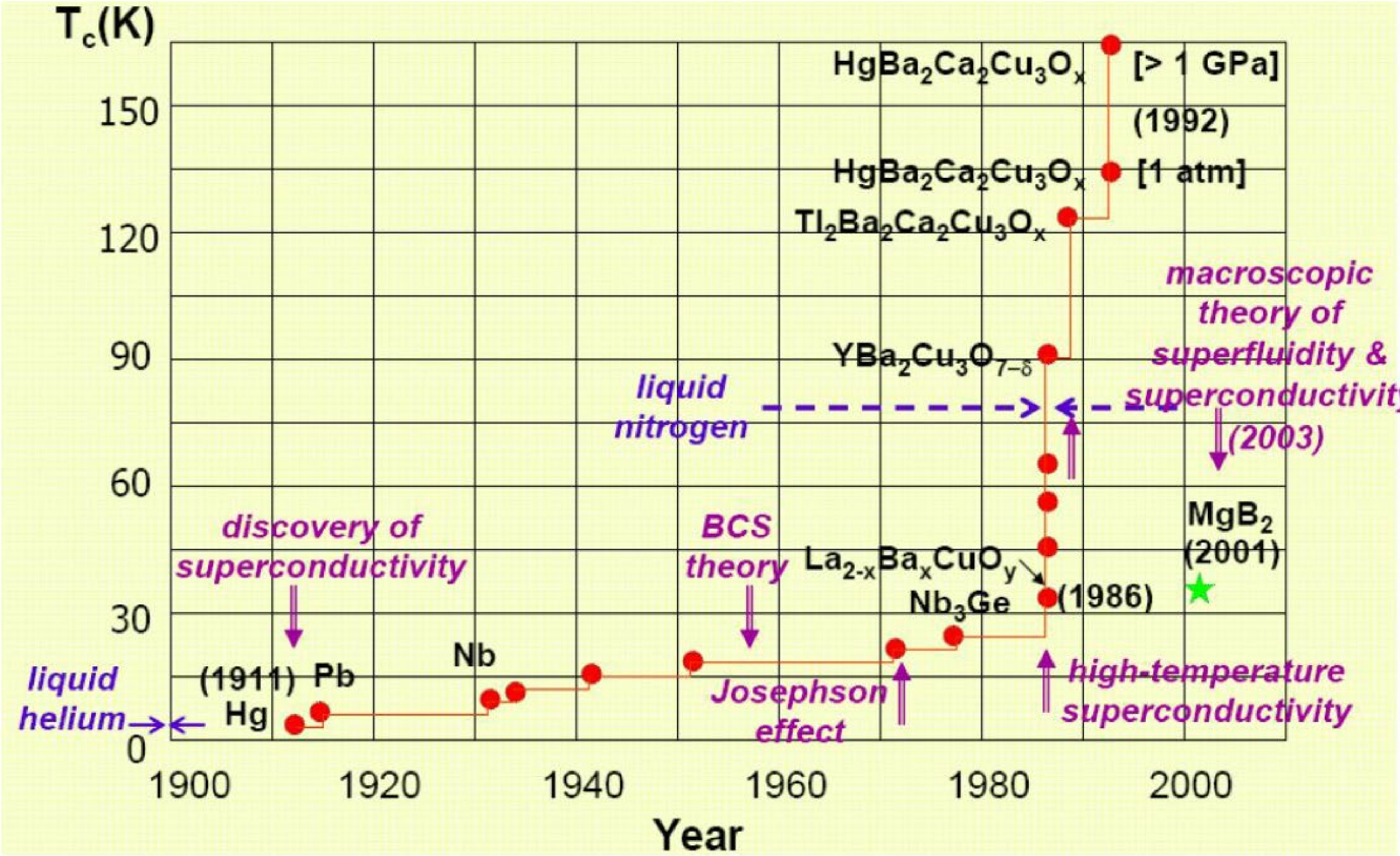
Surface treatments for State of the art SRF cavities

- SRF is highly efficient but complex technology
- Supercurrents only flow within a few tens of nanometres
  - Performance is very sensitive to near surface material properties which can be engineered by heat treatments in vacuum or low pressure gas atmosphere
- Maximum quality factor and accelerating gradient depend on surface treatment but also on RF frequency, cavity shape (surface field configuration), ambient magnetic flux in a correlated and not fully understood way





Superconducting Materials



Outline

- Quick recap of London theory and demonstration of the Meissner effect
- Surface Resistance
  - Electrodynamics of normal conductors
    - Normal and anomalous skin effect
  - Electrodynamics of superconductors
    - Surface impedance of superconductors in the two fluid model and the BCS theory
    - Residual resistance
    - Field dependence of surface resistance
- Maximum RF field
  - DC critical fields, Hc, Hc1, Hc2, Hsh
  - Critical field under RF
- Materials for SRF
  - Why niobium
  - Materials beyond niobium
  - Multilayers

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The London Theory

Assume an electron which is freely accelerated by an electric field

Lorentz force acting on the particle:

$$\boldsymbol{F} = m\boldsymbol{a} = \boldsymbol{m}\frac{\partial \boldsymbol{v}}{\partial t} = -e\boldsymbol{E} \quad \Rightarrow \quad \frac{\partial \boldsymbol{J}}{\partial t} = \frac{n_S e^2}{m}\boldsymbol{E}$$

$\boldsymbol{J} = -e n_S \boldsymbol{v}$

Definition of the current density

1st London Equation

To explain the Meissner effect we want to derive an equation that relates ***J*** to ***B***

Maxwell equation:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

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The goal is to obtain a differential equation for ***B***

The London Theory

Ampere´s law $\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}$ relates the current density ***J*** and the magnetic flux density ***B***

$$\frac{\partial}{\partial t}\left[\nabla \times \boldsymbol{J} + \frac{n_S e^2}{m}\boldsymbol{B}\right] = 0 \quad \xrightarrow{\mu_0 \nabla \times \boldsymbol{J} = -\nabla^2 \boldsymbol{B}} \quad \frac{\partial}{\partial t}\left[\nabla^2 \boldsymbol{B} - \frac{\mu_0 n_S e^2}{m}\boldsymbol{B}\right] = 0$$

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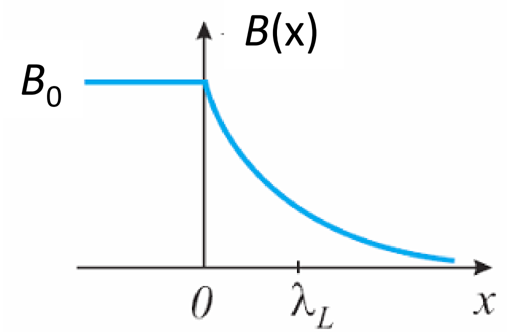

2 possible solutions:

1. $B = \text{const}$
2. $B(x) = B_0 \exp(-\frac{x}{\lambda_L})$ with $\lambda_L = \sqrt{\frac{m}{\mu_0 n_S e^2}}$ London penetration depth

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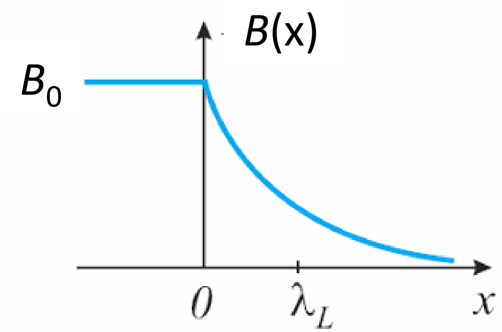

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Let us check which solution is physically meaningful

Perfect Conductor

$$\frac{\partial}{\partial t}\left[\nabla^2 \boldsymbol{B} - \frac{\mu_0 n_S e^2}{m}\boldsymbol{B}\right] = 0$$

Consider a conductor which fulfills these solutions when cooled below its critical temperature

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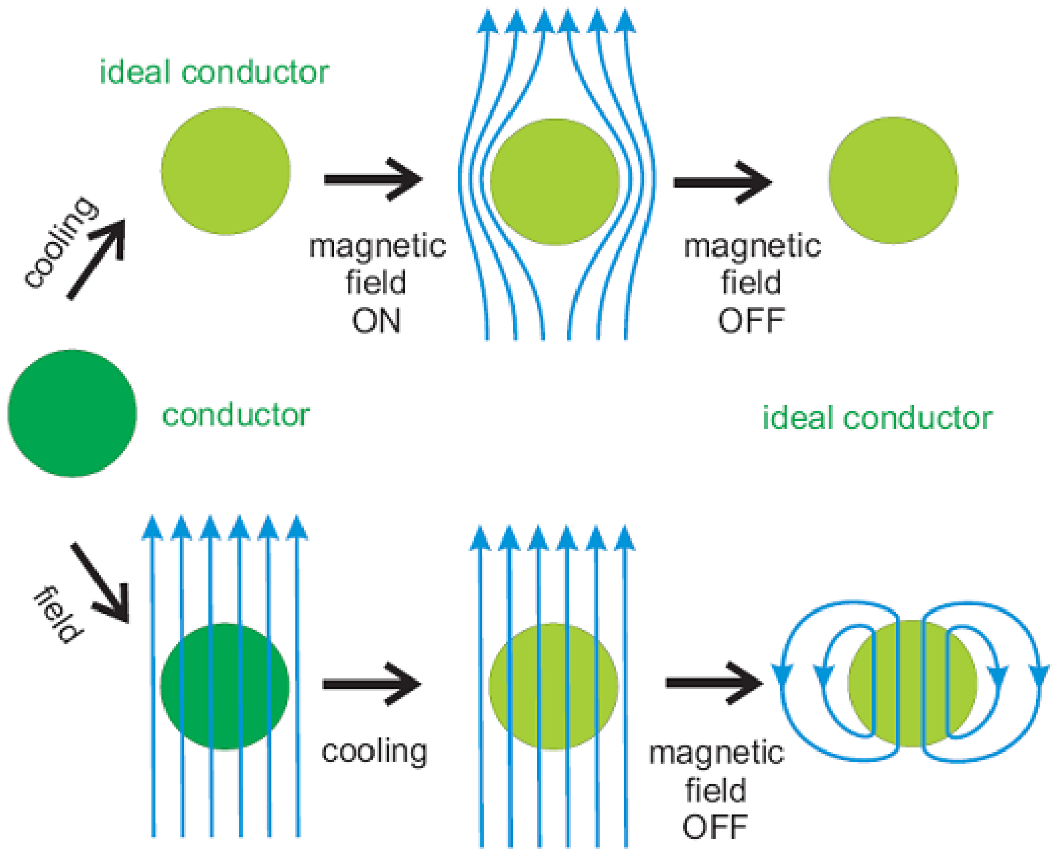

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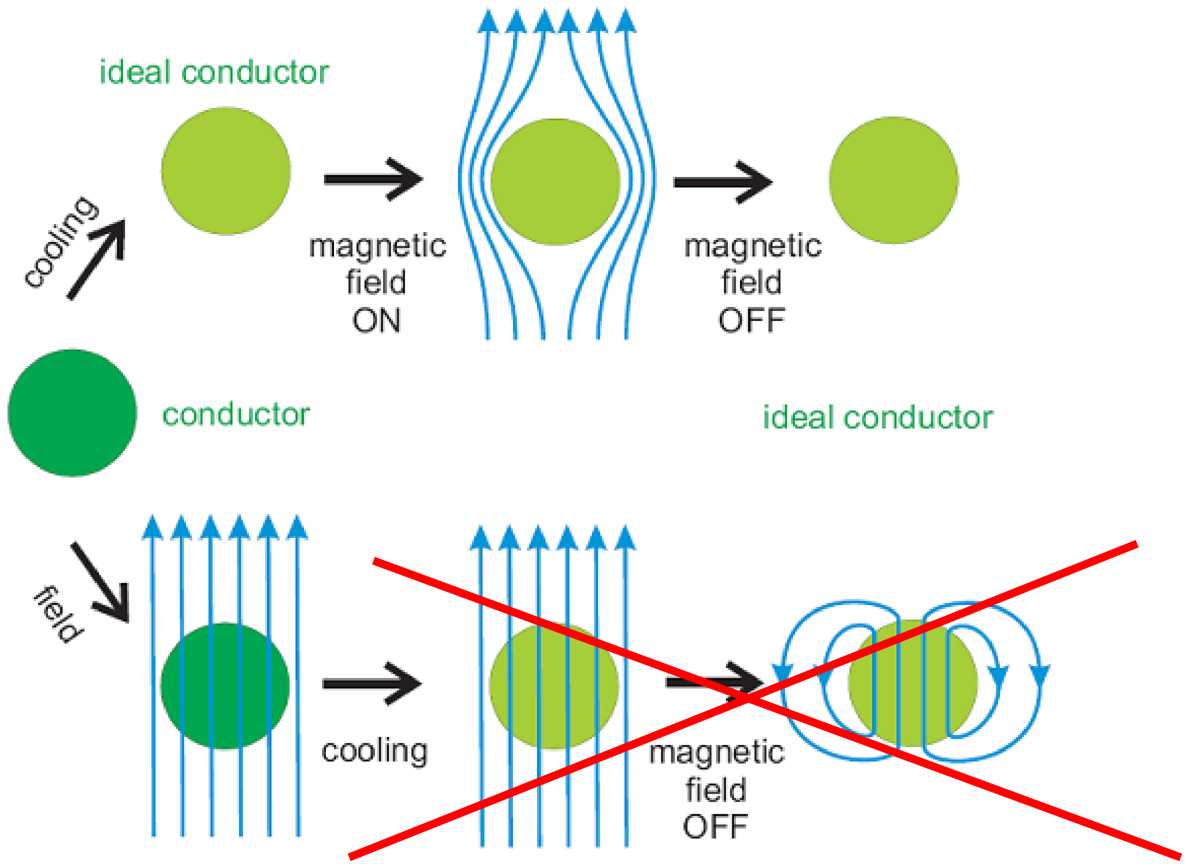

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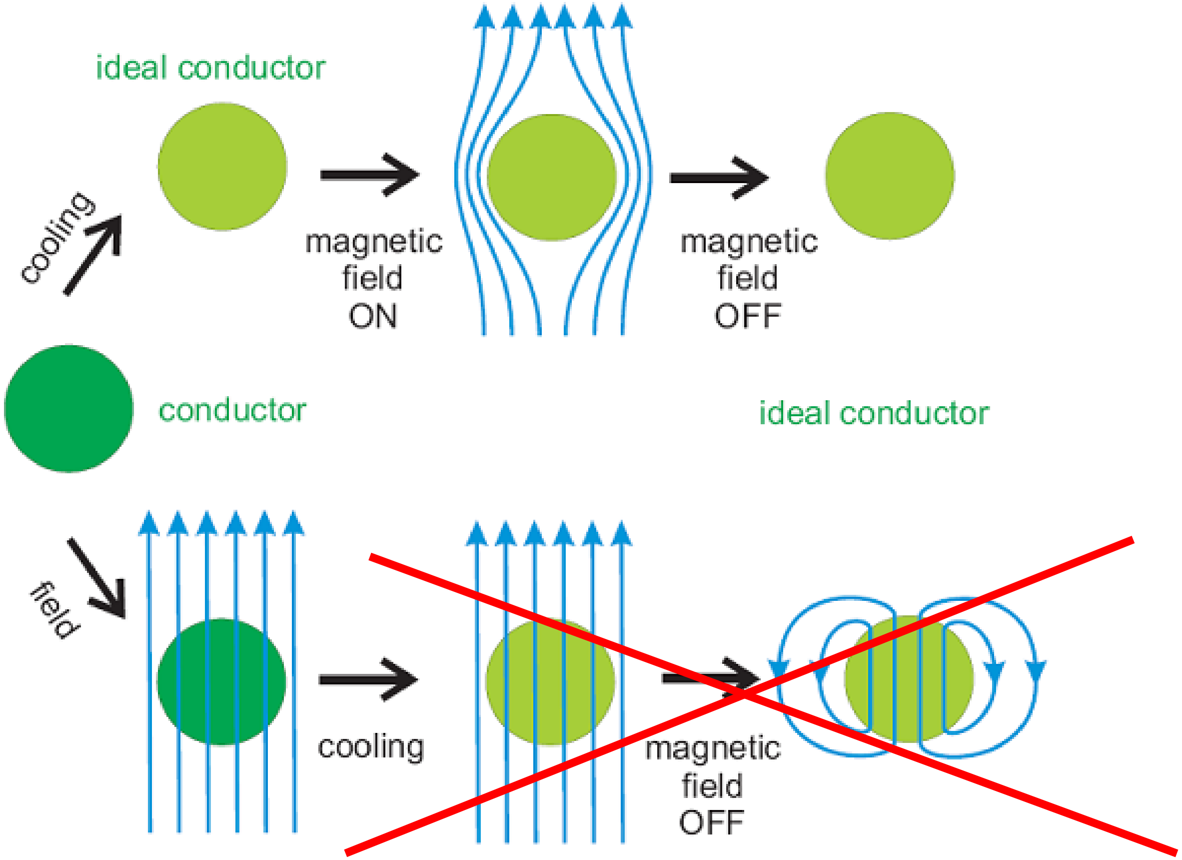

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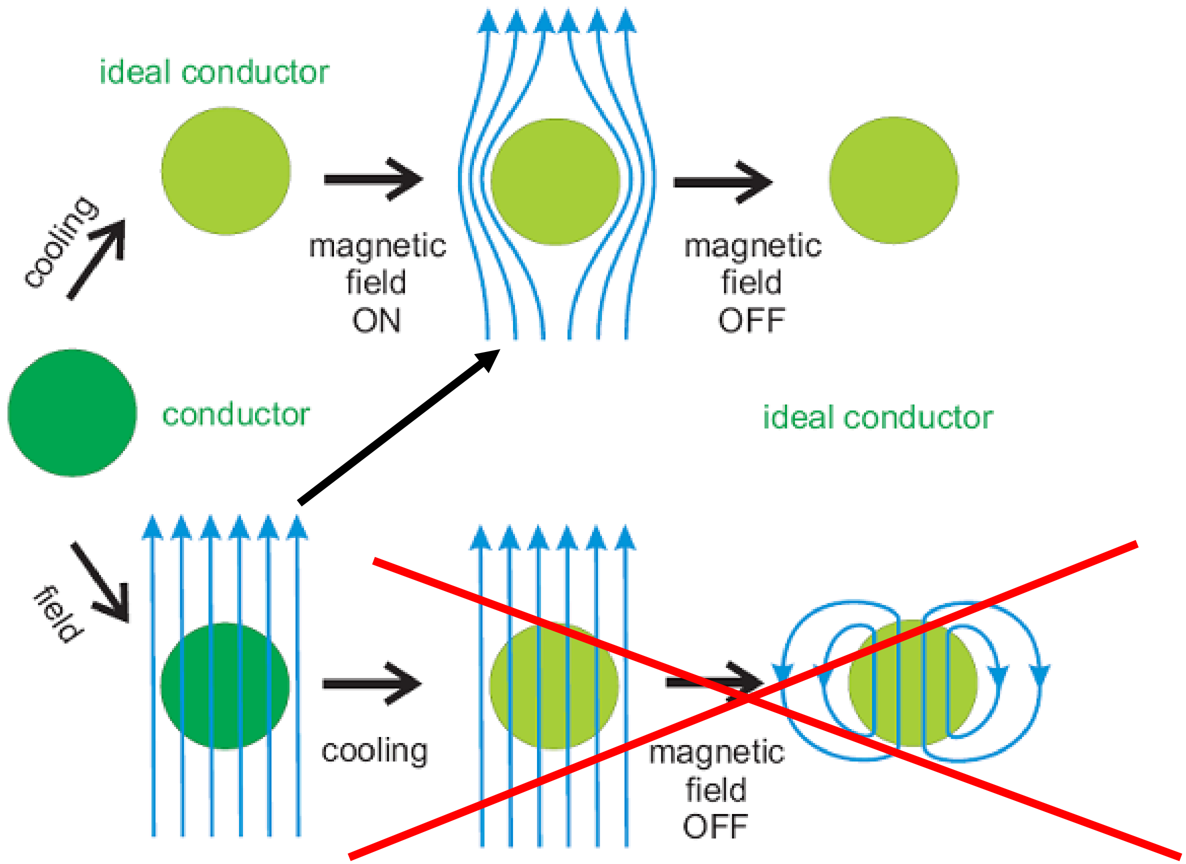

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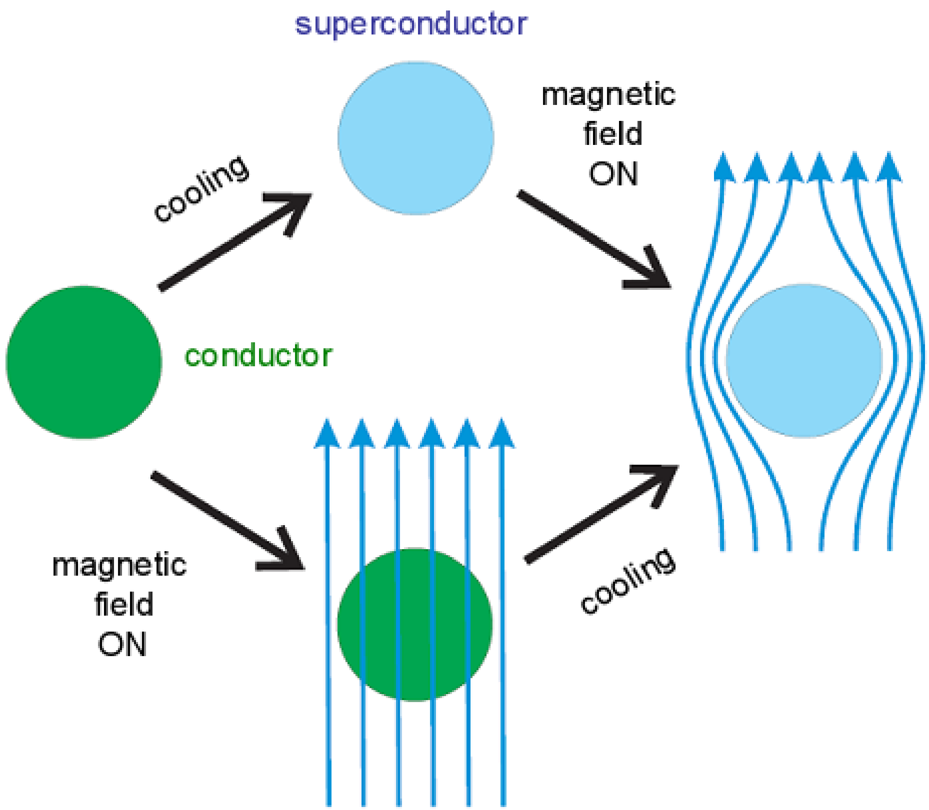
Superconductor - Meissner Effect



~~1. $B = \text{const}$~~

2. $B(x) = B_0 \exp(-\frac{x}{\lambda_L})$

- Superconductivity is a phase transition
- The final state does not depend on the order of cooling and applying field
- The constant solution is not physically meaningful

London Theory and Meissner Effect

Only exponential decaying fields are observed

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To explain the Meissner effect the Londons postulated:

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Finally we have derived the two London equations:

$$\frac{\partial \boldsymbol{J}}{\partial t} = \frac{n_S e^2}{m}\boldsymbol{E}$$

Zero Resistance

$$\nabla \times \boldsymbol{J} = -\frac{n_S e^2}{m}\boldsymbol{B}$$

Meissner Effect

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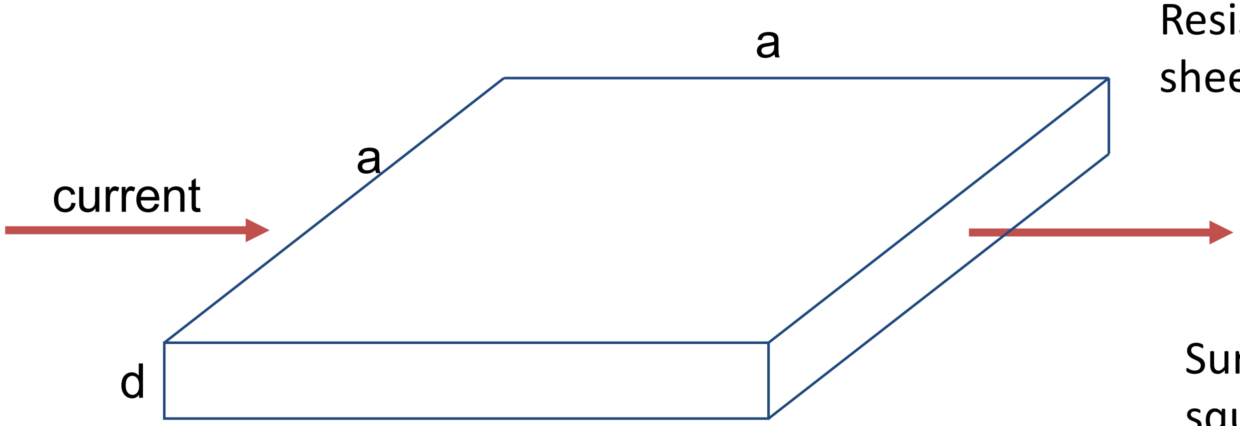
Surface resistance: intuitive meaning

Since we will deal a lot with the surface resistance R_s in the following, here is a simple **DC model that gives a rough idea** of what it means:

Consider a square sheet of metal with resistivity ρ and calculate its resistance to a transverse current flow:



Courtesy: S. Calatroni

Resistance of a square metal sheet of thickness d:

$$R = \frac{\rho\, \mathrm{a}}{\mathrm{d}\, \mathrm{a}}$$

Surface Resistance of a square metal sheet with penetration depth δ:

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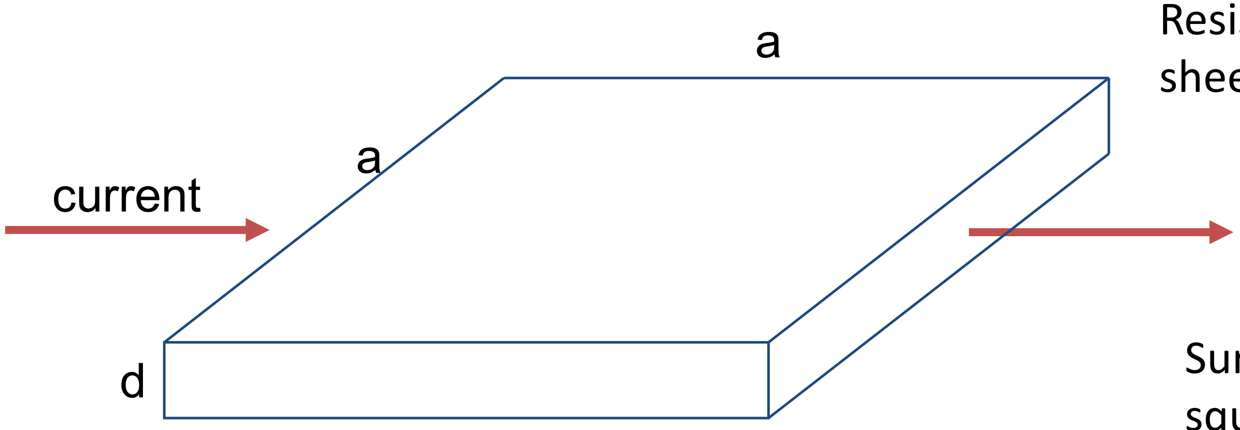

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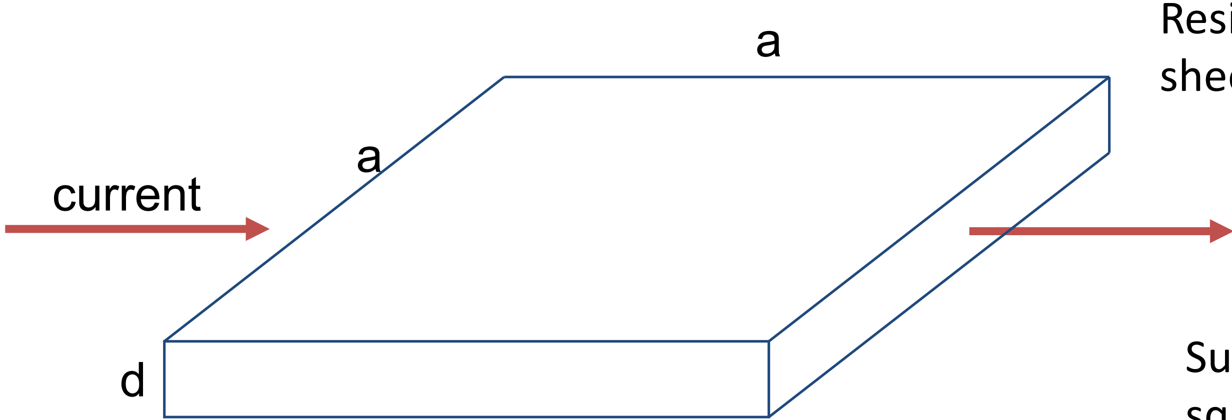

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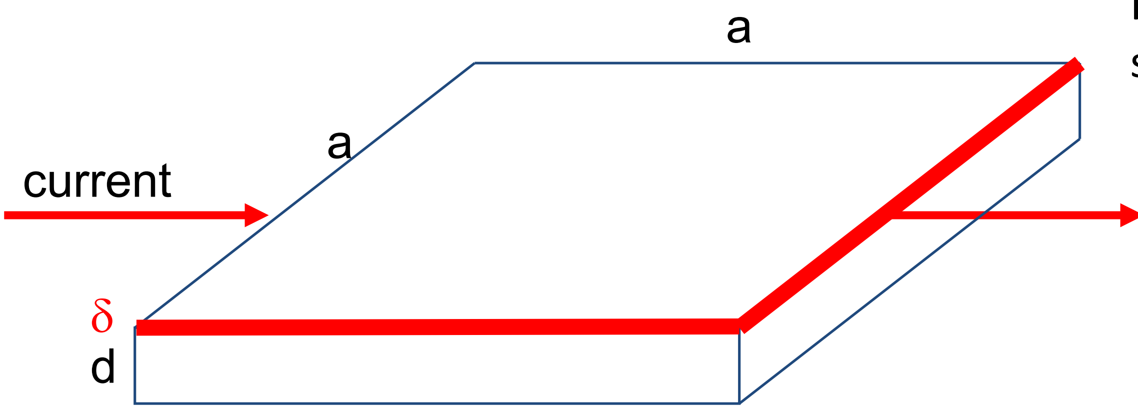

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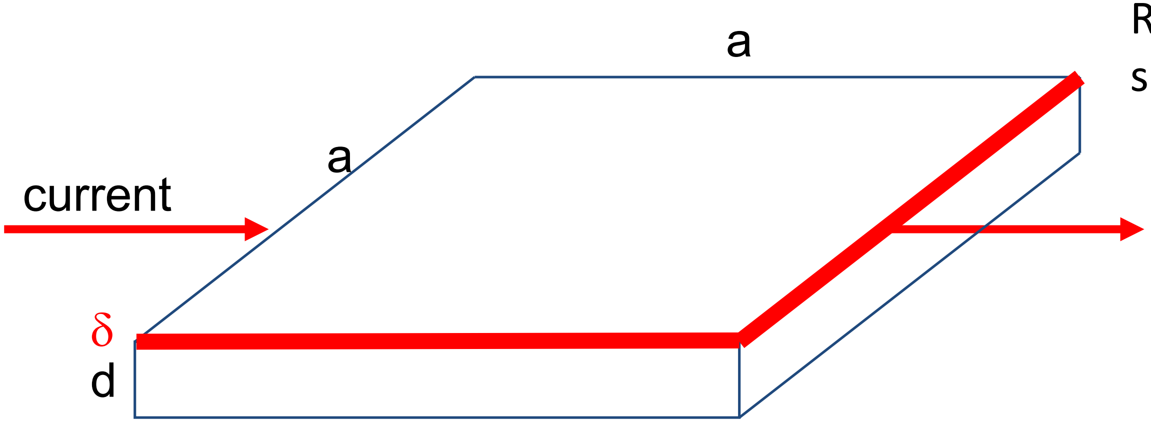

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$$R = \frac{\rho \not{a}}{d \not{a}} = \frac{\rho}{d}$$

Surface Resistance of a square metal sheet with penetration depth δ:

$$R_s = \frac{\rho}{\delta}$$

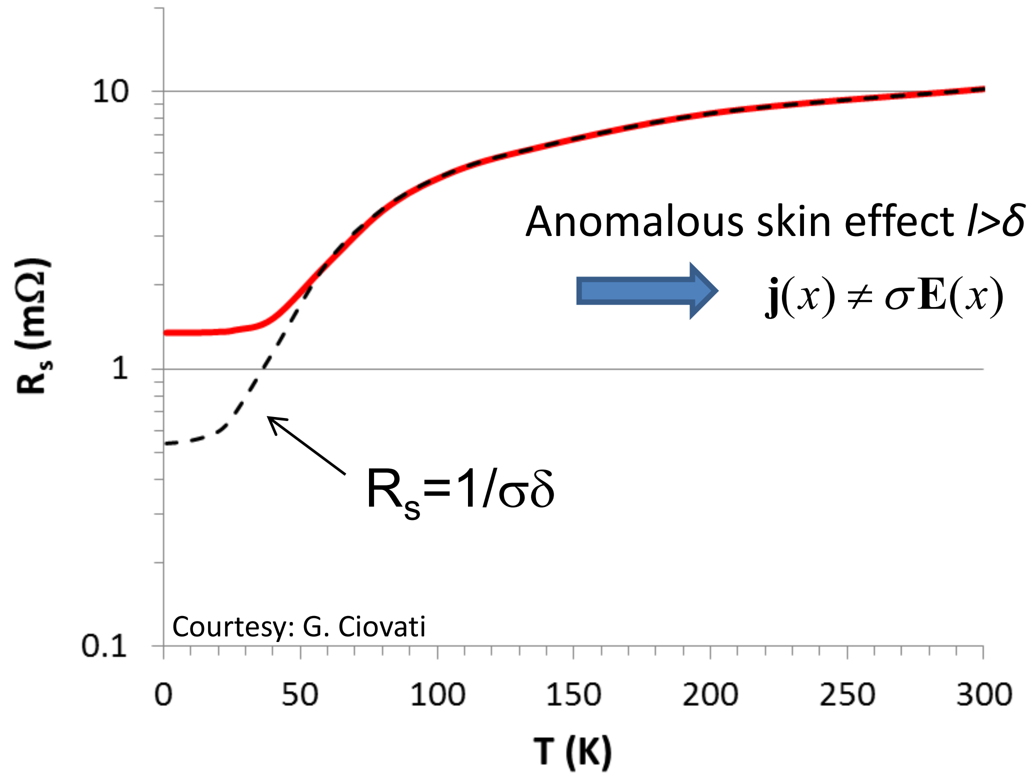
In this model the surface resistance R_s is the resistance that a square piece of conductor opposes to the flow of the currents induced by the RF wave, within a layer δ

What happens at low temperature?

Surface resistance of Cu at 1.5 GHz as a function of temperature with conductivity $\sigma=1/\rho$

$$R_s = X_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\mu_0\mu\omega}{2\sigma}}$$

- At room temperature the conductivity is dominated by phonon scattering
- At low temperature phonons "freeze out" and the conductivity depends on impurity concentration.
- The residual resistivity ratio **RRR = σ(0K)/σ(300K)** is a measure of material purity



...in spite of the resistivity decreasing by a factor 300 from 300 K to 4.2 K, R_s only decreases by a factor of ~8!

To reduce R_S below the mΩ range for RF application we need superconductivity!

Surface Resistance of Superconductors

- Superconducting currents are transported by Cooper pairs formed of two electrons
  - flow without friction → DC supercurrents are lossless
- For temperatures above 0 K not all electrons form Cooper pairs
- Cooper pairs have a finite inertia. Under RF fields a time-varying E-field is induced in the material. Normal electrons see this field, move and dissipate

$$n_n(T) \propto e^{-\Delta/k_B T}$$

+

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

+

$j = \sigma E$

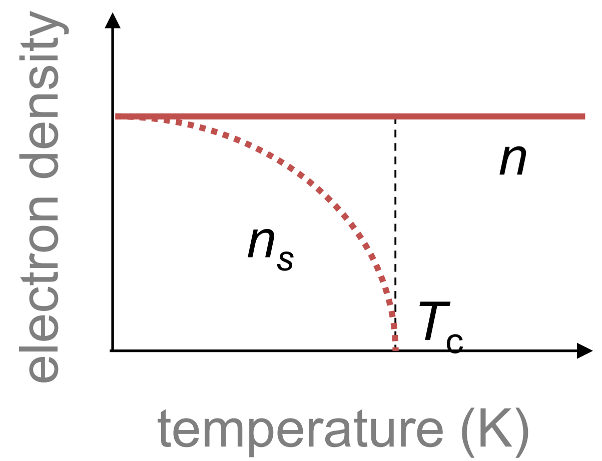
⬇

$R_s > 0$

Surface Resistance in the two fluid model

Basic ingredients for RF superconductivity

- Two fluid model (Gorter-Casimir)
- Maxwell electrodynamics
- London equations



Basic assumptions of two fluid model

- all free electrons of the superconductor are divided into two groups:
  - superconducting electrons of density n_s
  - normal electrons of density n_n
- The total density of the free electrons is
  $n = n_s + n_n$
- As the temperature increases from 0 to T_c, the density n_s decreases from n to 0.

$$n_n = \exp(-\frac{\Delta}{k_b T})$$

Surface Resistance in the two fluid model

London equation:

$$\frac{\partial}{\partial t}\vec{J}_s = \frac{\vec{E}}{\mu_0 \lambda_L^2} \quad \Rightarrow \quad J_s = -i\frac{1}{\omega\mu_0\lambda_L^2}E$$

$$E = E_0 e^{i\omega t}$$

$$\frac{1}{\omega\mu_0\lambda_L^2} = \sigma_2$$

Two fluid model:

$$\Rightarrow \quad J = J_n + J_s = \underbrace{(\sigma_1 - i\sigma_2)}_{\sigma}E$$

$$\sigma_1 = \frac{n_n e^2 \tau}{m}, \quad \sigma_2 = \frac{n_s e^2}{m\omega}$$

- Electrodynamics of sc is the same as nc, only that we have to change $\sigma \rightarrow \sigma_1 - i\,\sigma_2$

- Penetration depth: $\delta = \sqrt{\frac{2}{\mu_0\sigma\omega}} = \frac{1}{\sqrt{\mu_0\omega\sigma_2}}\sqrt{\frac{2i}{1+i\sigma_1/\sigma_2}} \cong (1+i)\lambda_L\left(1-i\frac{\sigma_1}{2\sigma_2}\right)$

$\sigma_1 << \sigma_2$ for sc at $T<<T_c$

Scattering time $\tau = l/v_F \approx 10^{-14}$s

Surface Resistance in the two fluid model

London equation:

$$\frac{\partial}{\partial t}\vec{J}_s = \frac{\vec{E}}{\mu_0 \lambda_L^2} \Rightarrow J_s = -i\frac{1}{\omega\mu_0\lambda_L^2}E$$

$$E = E_0 e^{i\omega t}$$

$= \sigma_2$

Two fluid model:

$$\Rightarrow J = J_n + J_s = (\sigma_1 - i\sigma_2)E$$

σ

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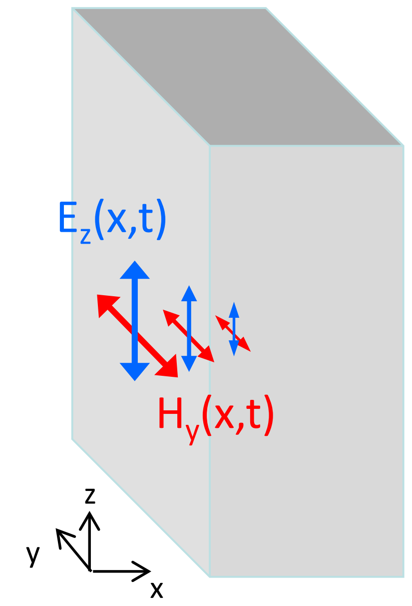
$\sigma_1 \ll \sigma_2$ for sc at $T \ll T_c$

Scattering time $\tau = l/v_F \approx 10^{-14}$s

Interesting to note here: -We have derived λ_L from DC arguments before
- Now we find $\delta=\lambda_L$ for $T \ll T_c$

Surface Resistance in the two fluid model

- Electrodynamics of sc is the same as nc, only that we have to change $\sigma \rightarrow \sigma_1 + i\,\sigma_2$
- Recall the definition of the surface impedance:



$$Z = \frac{|E_{||}|}{\int_0^{\infty} J(x)dx} = \frac{E_{||}}{H_{||}} = R_s + i\,X_s = \sqrt{\frac{i\omega\mu_0}{\sigma}}$$

$$\sigma = \sigma_1 + i\sigma_2 \qquad \sigma_1 = \frac{n_n e^2 \tau}{m}, \quad \sigma_2 = \frac{n_s e^2}{m\omega}$$

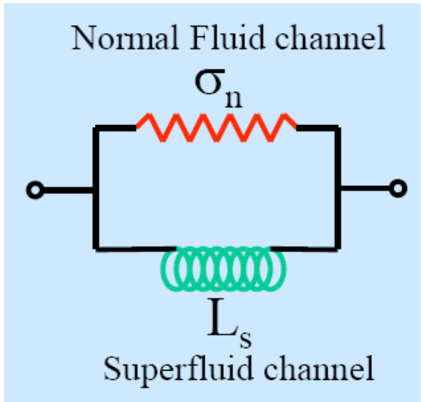
- For $\sigma_1 << \sigma_2$ we obtain:

$$X_s = \omega \underbrace{\mu_0 \lambda_L}_{L_s}$$

L_s: kinetic inductance

$$Z_s = R_s + i\,X_s$$

$$R_s = \frac{1}{2}\mu_0^2 \omega^2 \sigma_1 \lambda_L^3$$



Rs within BCS theory

- Mattis and Bardeen (1958) used time dependent perturbation theory to derive R_S for weak RF fields
- Within this theory no simple formula can be derived. Several approximate formula can be found in the literature for some limits. For example for the dirty limit

$$R_S = \frac{1}{2}\omega^2\lambda^3\sigma_1\mu_0^2 \ln\left(\frac{\Delta}{\hbar\omega}\right) \exp\left(-\frac{\Delta}{k_b T}\right)/T$$

- There are numerical codes (Halbritter (1970) to calculate R_{BCS} as a function of ω, T and material parameters (ξ_0, λ_L, T_c, Δ, l)
- For example, http://www.lepp.cornell.edu/~liepe/webpage/researchsrimp.html

Rs within BCS theory

SRIMP

This webpage calculates BCS surface resistance under wide range of conditions, and is based on a program by Jurgen Halbritter. [J. Halbritter, Zeitschrift f�r Physik 238 (1970) 466]

Enter material parameters below, and click submit to calculate the BCS surface resistance. Results are given in a new window.
Please be aware that frequencies much lower than 1 MHz may cause substantial processing times (depending on the user's computer).

Submit

Frequency (MHz):	1300
Transition temperature (K):	9.2
DELTA/kTc:	1.86
London penetration depth (A):	330
Coherence length (A):	400
RRR:	300
Accuracy of computation:	.001
Temperature (of operation):	2

Be carefureful here. The website suggests 40nm. The input required is $\pi\xi_0/2$, while $\xi_0 \approx 40$nm for Nb

Results:

Diffuse Reflection:	Resistance (Ohm):	1.9031321344341478e-8
	Penetration Depth (um):	0.037746828693838295

Input Parameters:

Frequency (MHz):	1300
Transition temperature (K):	9.2
DELTA/kTc:	1.86
London penetration depth (A):	330
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BCS vs two fluid model

- The treatment within BCS theory and two-fluid model give qualitatively similar results
- Quantitatively they can differ by an order of magnitude
  - The BCS treatment gives qualitatively correct results for low field
- To treat experimental data approximate formulae are useful, e.g.

$$R_{\mathrm{S}} = \frac{\omega^2 \mathrm{A}}{T} \exp\left(-\frac{\Delta}{k_b T}\right)$$

- Here *A* accounts for all material parameters

The RF surface resistance

$$R_{\mathrm{BCS}} = \omega^2 \lambda^3 \sigma_1 \mu_0^2 \exp\left(-\frac{\Delta}{k_b T}\right)$$

This equation implies R_s:

- Has a minimum for medium purity
- Is proportional to ω^2
- Decreases exponentially with temperature
- Vanishes as $T \rightarrow 0$ K
- Is independent of RF field strength

In the following we will compare these assumptions to experimental data and modify the formula if necessary

Material purity dependence of R_s

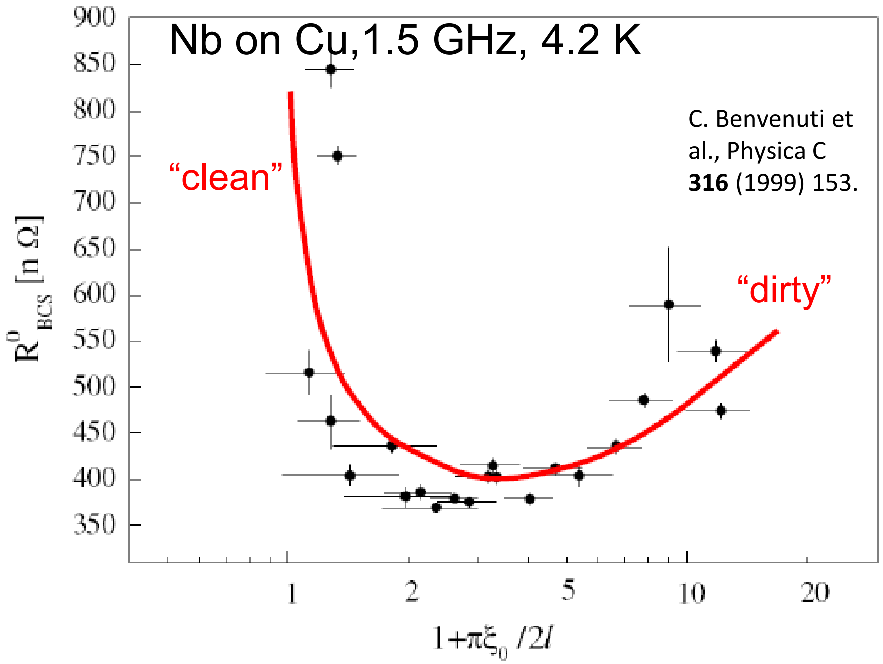
$$R_{\text{BCS}} = \omega^2 \lambda^3 \sigma_1 \mu_0^2 \exp\left(-\frac{\Delta}{k_b T}\right)$$

- The dependence of the penetration depth on l is approximated as $\lambda(l) \approx \lambda_L \sqrt{1 + \frac{\pi \xi_0}{2l}}$
- $\sigma_1 \propto l$

$$\Rightarrow R_S \propto \left(1 + \frac{\pi \xi_0}{l}\right)^{3/2} l \Rightarrow$$

$R_s \propto l$ if $l >> \xi_0$ ("clean" limit)

$R_s \propto l^{-1/2}$ if $l << \xi_0$ ("dirty" limit)



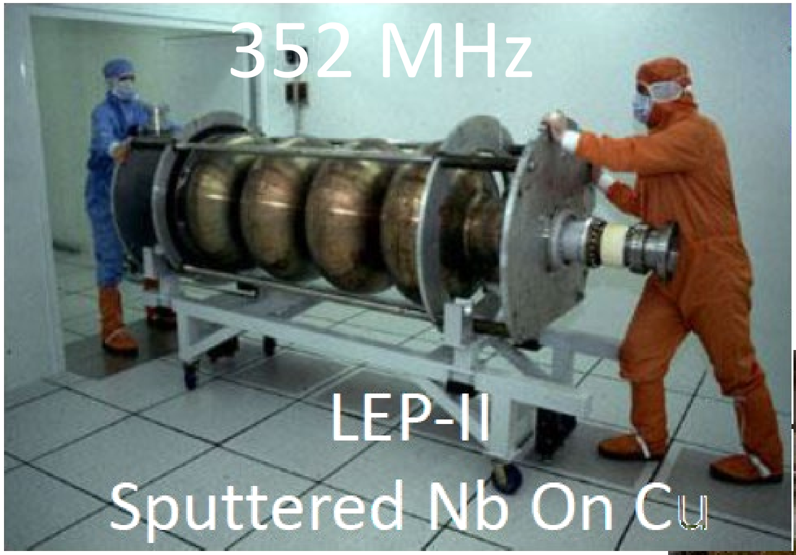
R_s has a minimum for $l = \pi\xi_0/4$

Example: Nb films sputtered on Cu substrate

- By changing the sputtering species, the mean free path was varied
- RRR of niobium on copper cavities can be tuned for lowest R_S.

CERN – Nb on Cu cavities

CERN first started to use Nb on Cu technology for LEP-II cavities.
Du to the low frequency and optimal mean free path economical operation at 4.5K was possible



The technology was then adopted for the 400 MHz LHC cavities

Hie-Isolde Quarter Wave Resonator commissioned in 2015