Fundamentals of superconductivity

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International Accelerator School: Superconducting Science and Technology for particle accelerators

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Outline

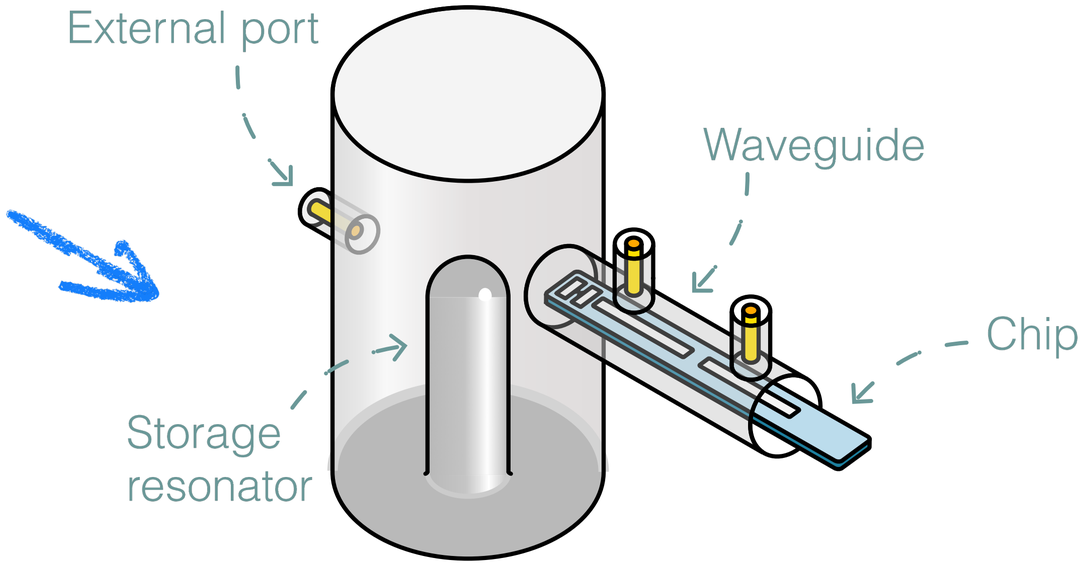
1. Survey of superconductivity
2. Microscopic origin: A survey of Cooper pairs and BCS theory
3. Macroscopic quantum behaviour and Ginzburg-Landau theory
4. Derivation of London equations and Meissner effect
5. Penetration depth and inductance
6. Critical fields and superconductors of Types I and II
7. Application to cavities: Quality factor and surface resistance due to quasiparticles

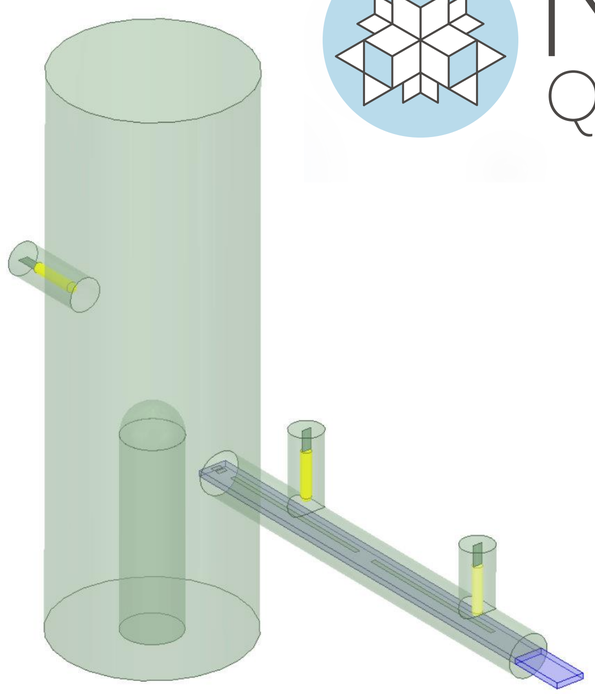
Why superconducting cavities? Ans: Extremely low loss.

Useful for quantum computing in the low energy regime, and for particle accelerators in the high energy regime

- Company Nord Quantique based in Sherbrooke - QC has as its main mission the integration of 3d cavities to superconducting circuit chips.

Why using 3D architecture?

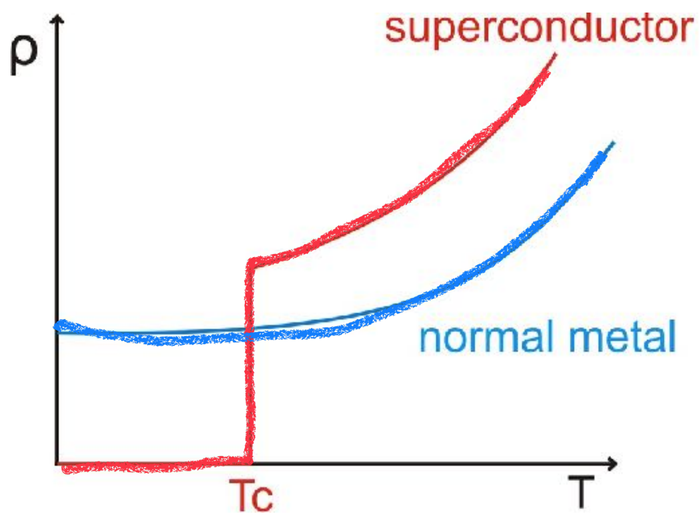




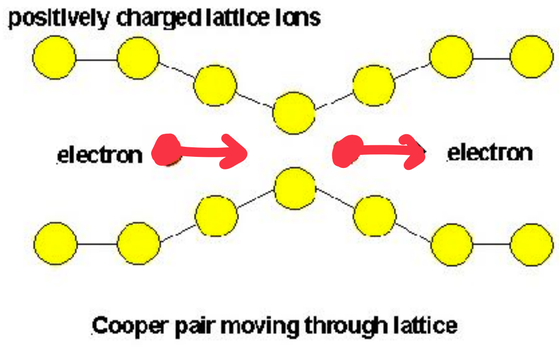
- Cavity with highest lifetime in the industry (~30 ms)

Survey of superconductivity

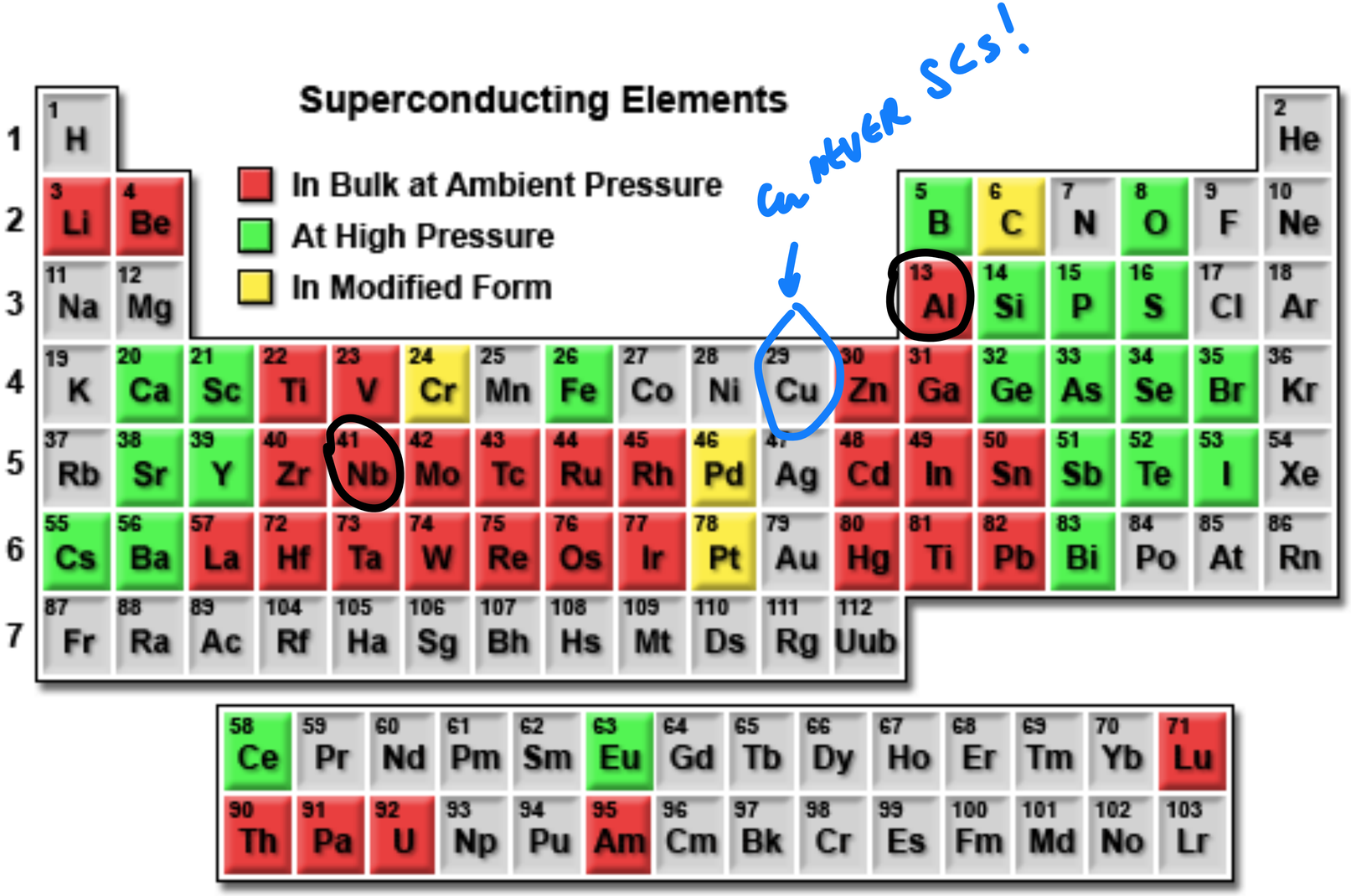
- You're probably familiar with the fact that resistance drops to zero when temperature T decreases below the superconducting critical temperature T_c:



- The microscopic explanation for this was given by Bardeen, Cooper, and Schrieffer (BCS): For some materials, electron-phonon interaction leads to attraction between electrons close to the Fermi surface. These form 2-electron "molecules" called Cooper pairs which move without resistance.



Superconducting materials



https://nationalmaglab.org/education/magnet-academy/learn-the-basics/stories/superconductivity-101

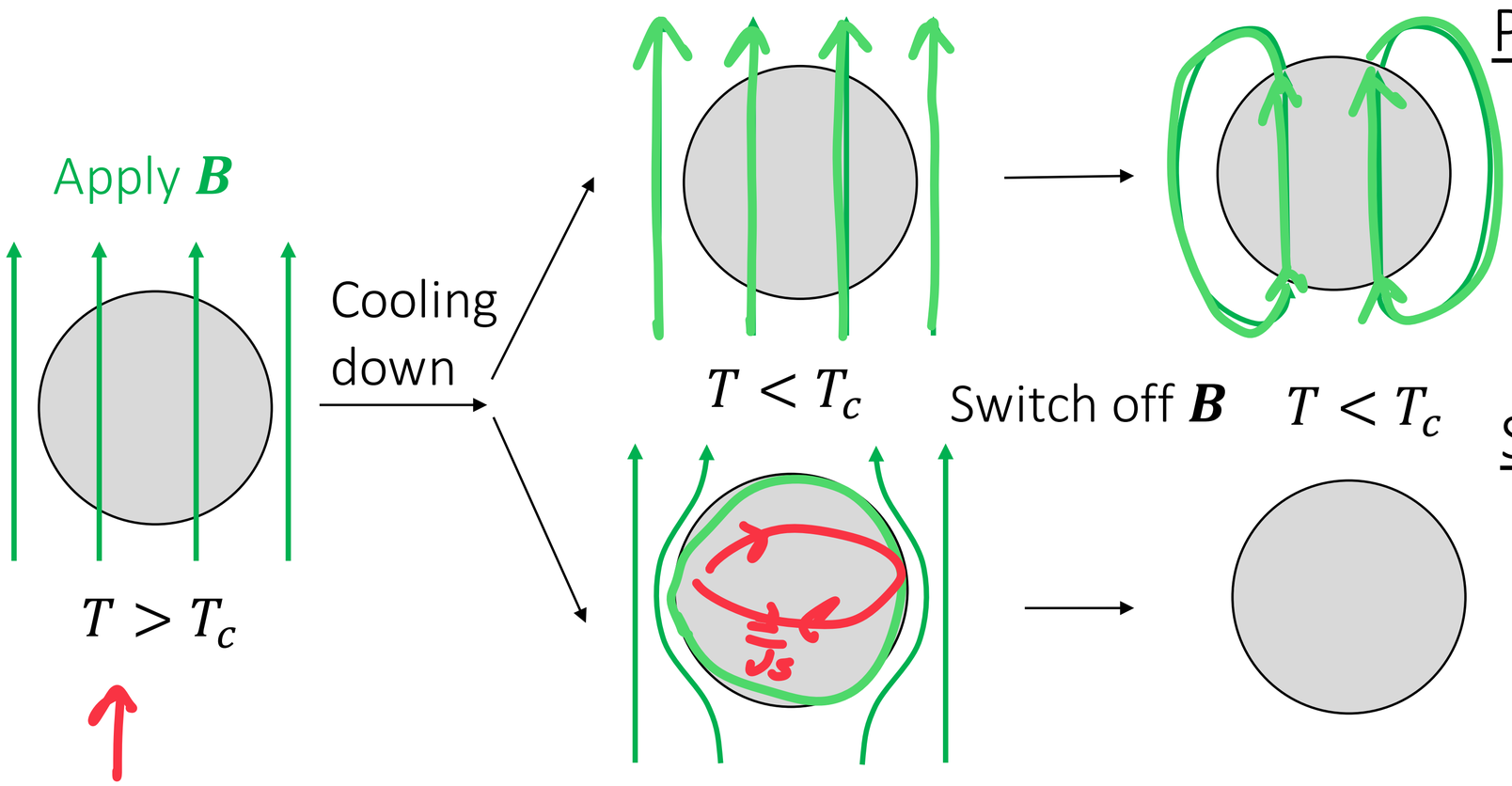
VALUES OF T_c AND H_c FOR THE SUPERCONDUCTING ELEMENTS[a]

ELEMENT		T_c (K)	H_c (GAUSS)[b]
Al		1.196	99
Cd		0.56	30
Ga		1.091	51
Hf		0.09	—
Hg	α (rhomb)	4.15	411
	β	3.95	339
In		3.40	293
Ir		0.14	19
La	α (hcp)	4.9	798
	β (fcc)	6.06	1096
Mo		0.92	98
Nb		9.26	1980
Os		0.655	65
Pa		1.4	—
Pb		7.19	803
Re		1.698	198
Ru		0.49	66
Sn		3.72	305
Ta		4.48	830
Tc		7.77	1410
Th		1.368	162
Ti		0.39	100
Tl		2.39	171
U	α	0.68	—
	γ	1.80	—

But… Zero resistance is not the whole story

Something remarkable happens in the SC state: Meissner effect

- A SC is qualitatively different from a normal metal whose resistance goes to zero at $T<T_c$:



Perfect Electric Conductor

$$\nabla^2\left(\frac{\partial \boldsymbol{B}}{\partial t}\right) - \frac{1}{\lambda_L^2}\left(\frac{\partial \boldsymbol{B}}{\partial t}\right) = 0$$

Preserve initial condition!

Superconductor

Zero field!

$$\nabla^2 \boldsymbol{B} - \frac{1}{\lambda_L^2}\boldsymbol{B} = 0$$

London equation

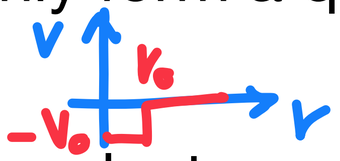
Additional constraint (broken Gauge symmetry)

A. Myiazaki, “Basics of RF superconductivity and Nb material” , SRF 2023 @Michigan State University

Microscopic origin of superconductivity

Cooper pairs: Two-electron bound states at the Fermi surface

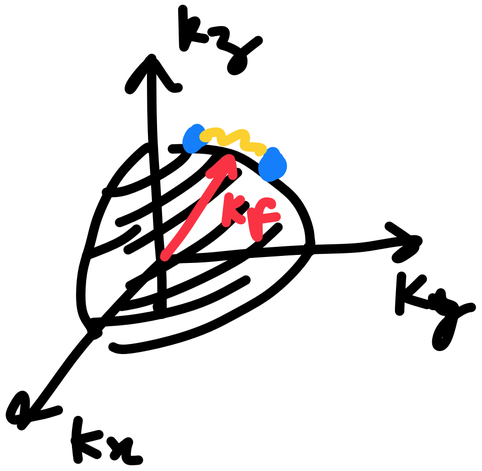
- Free particles in 3d: Can only form a quantum bound state in the presence of a <u>strong attractive</u> potential.



BOUND STATE ONLY WHEN $V_0 \gtrsim \frac{\hbar^2}{2m}\frac{1}{r_0^2}$

- Cooper, 1956: Consider two electrons in the Fermi surface of a Fermi gas. Even an <u>infinitesimally small attraction</u> forms a bound state!

$$E_k = \frac{\hbar^2}{2m^*}|\vec{k}|^2$$

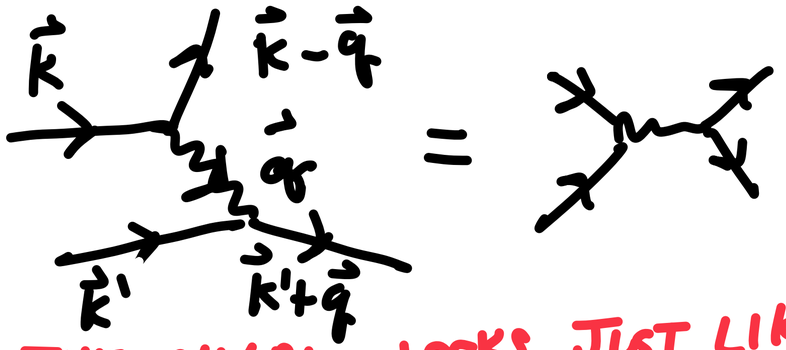


ELECTRONS ON FERMI SURFACE: EVEN $V_0 \to 0$ FORMS BOUND STATES!

BECAUSE STATES WITH $k < k_F$ ARE "BLOCKED" DUE TO PAULI EXCLUSION.

ORIGIN OF WEAK e-e ATTRACTION

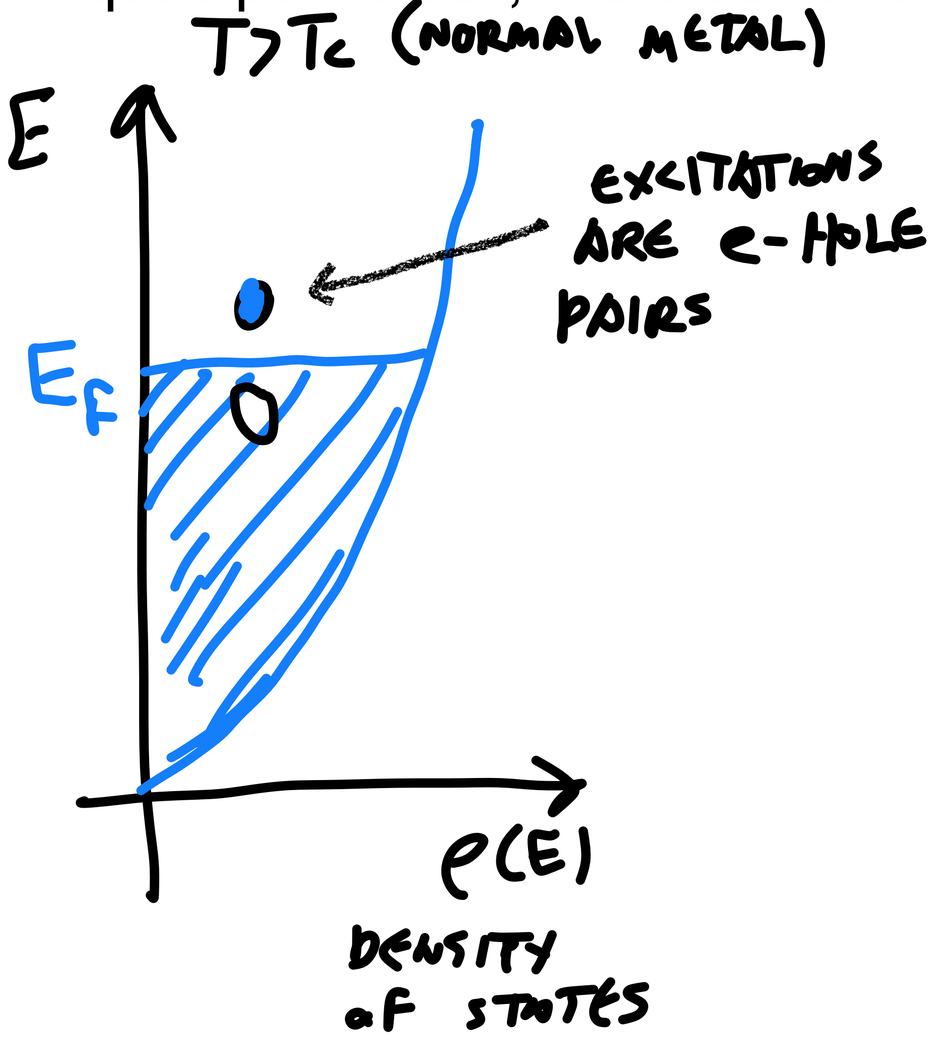
e-PHONON COUPLING:

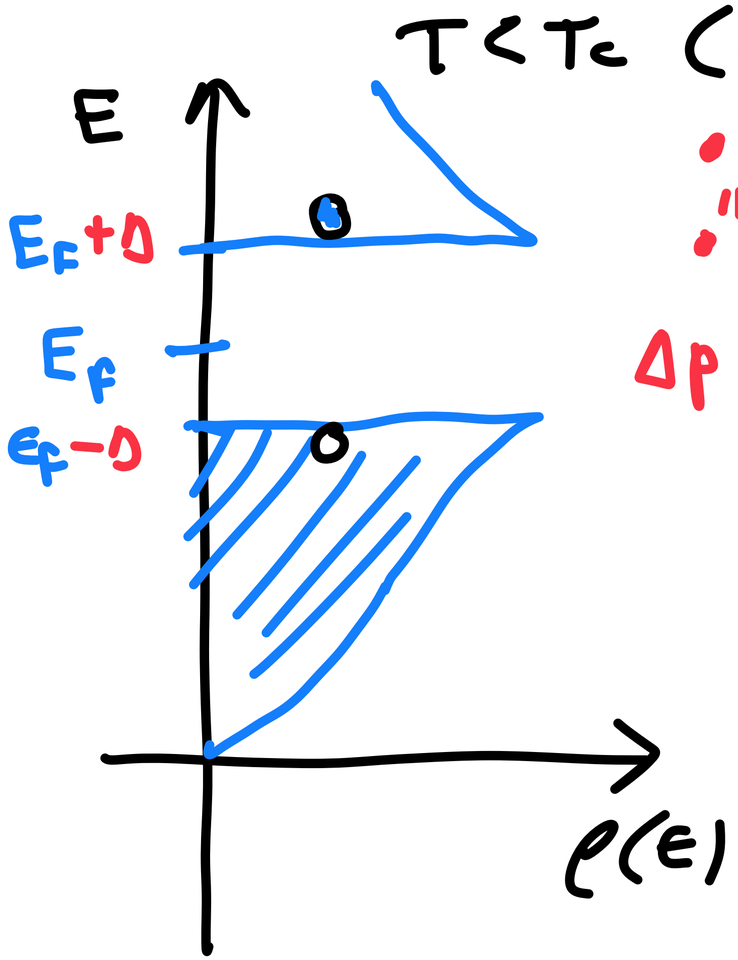


- THIS DIAGRAM LOOKS JUST LIKE e-e COULOMB REPULSION, WITH OPPOSITE SIGN.
- THIS e-e ATTRACTION IS STRONGER FOR SOME MATERIALS.

Bardeen-Cooper-Schrieffer (BCS) theory

- At $T<T_c$, an energy gap opens at the Fermi surface. Excited states are "quasiparticles", the electrons that result when a Cooper pair is "broken".





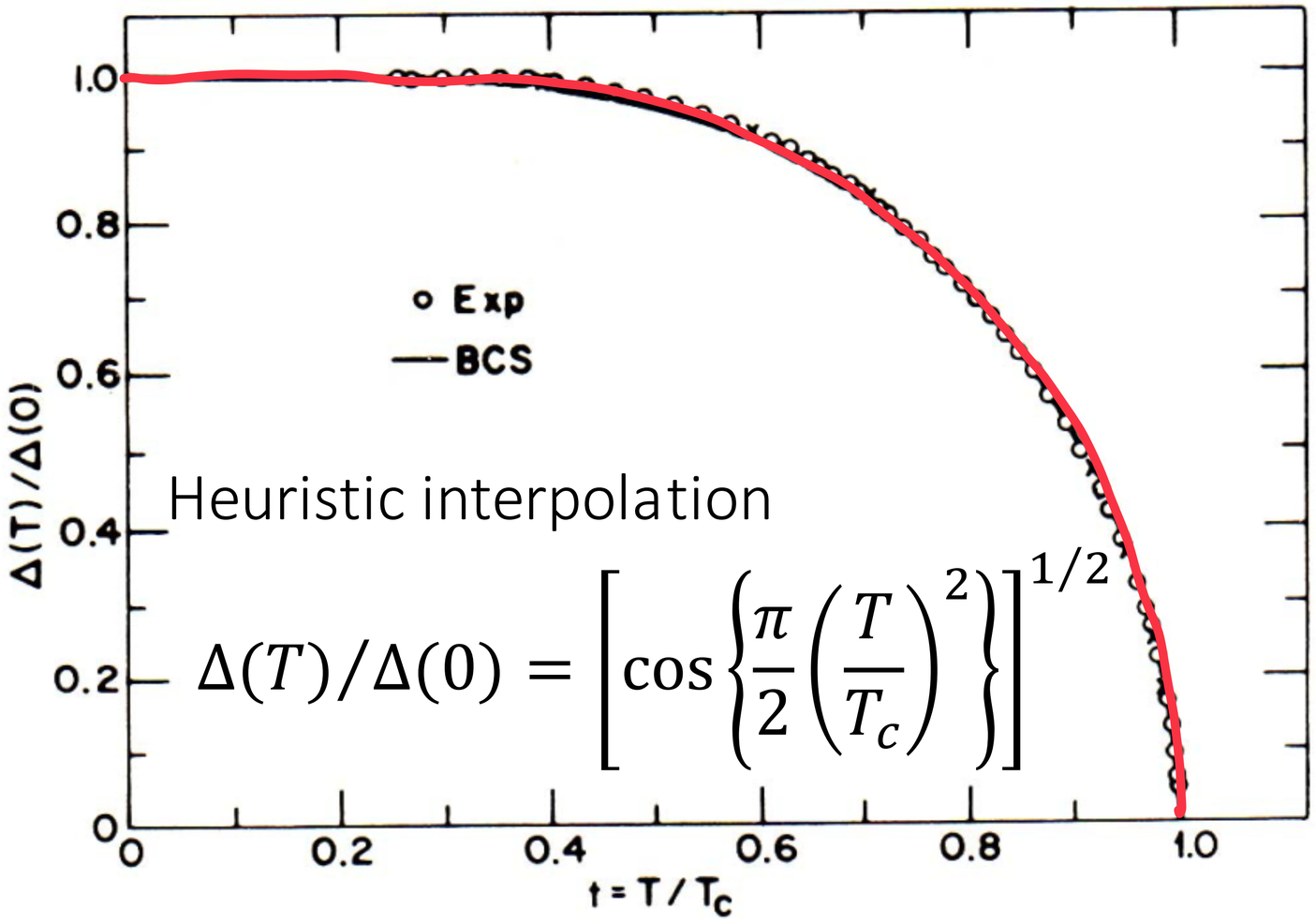
- SC GAP $\Delta \approx 1.76\, k_B T_c$
- "SIZE" OF COOPER

$$\Delta p \sim \frac{\Delta}{v_F} \Rightarrow \Delta x \sim \frac{\hbar}{\Delta p} = \frac{\hbar v_F}{\Delta} = \xi_0$$

ξ_0 "BARE" COHERENCE LENGTH.

ξ_0 IS SIZE OF PROB. DISTRIBUTION FOR e-e SPACING IN COOPER PAIR.

Dependence of SC energy gap on T/T_c



Heuristic interpolation

$$\Delta(T)/\Delta(0) = \left[\cos\left\{\frac{\pi}{2}\left(\frac{T}{T_c}\right)^2\right\}\right]^{1/2}$$

A. Myiazaki, “Basics of RF superconductivity and Nb material” , SRF 2023 @Michigan State University

Ginzburg-Landau theory (G-L): Macroscopic quantum behaviour

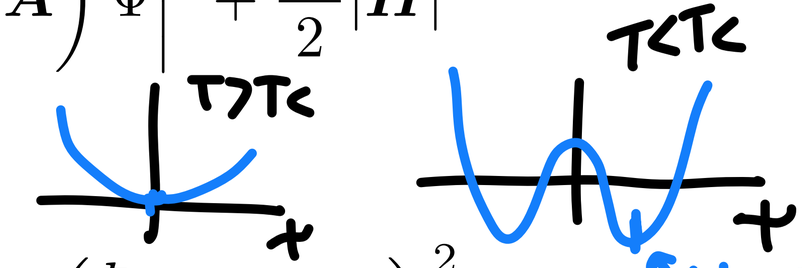
- The free energy of the SC state can be described by a complex order parameter Ψ(**r**):

FREE ENERGY OF NM

$$\mathcal{F} = \mathcal{F}_{\mathrm{N}} + \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \frac{1}{2m^*}\left|\left(\frac{\hbar}{i}\boldsymbol{\nabla} - e^*\boldsymbol{A}\right)\Psi\right|^2 + \frac{\mu_0}{2}|\boldsymbol{H}|^2$$

FREE ENERGY OF SC

$\alpha = \alpha'\left(\frac{T}{T_c} - 1\right) < 0 \quad T < T_c$



$|\psi_{eq}| > 0$!

- Minimize with respect to Ψ*(**r**):

$$\frac{\delta\mathcal{F}}{\delta\Psi^*} = \frac{\partial\mathcal{F}}{\partial\Psi^*} - \boldsymbol{\nabla}\cdot\frac{\partial\mathcal{F}}{\partial\boldsymbol{\nabla}\Psi^*} = \alpha\Psi + \beta|\Psi|^2\Psi + \frac{1}{2m^*}\left(\frac{\hbar}{i}\boldsymbol{\nabla} - e^*\boldsymbol{A}\right)^2\Psi = 0$$

JUST LIKE SCHRÖEDINGER'S EQN WITH $\mathcal{H} = \frac{1}{2m^*}\left(\frac{\hbar}{i}\vec{\nabla} - e^*\vec{A}\right)^2 + \underbrace{\beta|\psi|^2}_{\text{NONLINEAR POTENTIAL}}$ AND ENERGY $= -\alpha$!

$\mathcal{H}\psi = -\alpha\psi$

- These G-L eqns can be obtained from microscopic BCS theory. This shows Ψ(**r**) is the ensemble avg for the <u>centre of mass wave fnc</u> for Cooper pairs, and $e^*=2e$, $m^*=2m_e$.

Cooper pairs move without loosing coherence between each other

<u>ANALOGY</u>: NORMAL METAL ~ LIGHT BULB.
SC ~ LASER.

London eqns from G-L

- Interpret $|\Psi|^2$ as a density, obtain current: ASSUME $i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi$, CALCULATE $\frac{\partial|\psi|^2}{\partial t} = -\vec{\nabla}\cdot\vec{J}_s$

$$\boxed{\boldsymbol{J}_S = \frac{e^*\hbar}{m^*}\mathrm{Im}\left(\Psi^*\nabla\Psi\right) - \frac{e^{*2}}{m^*}\left|\Psi\right|^2\boldsymbol{A}}$$

JUST LIKE PROBABILITY CURRENT IN QM!

- Plug $\Psi(\boldsymbol{r}) = \sqrt{n_s}e^{i\theta(\boldsymbol{r})}$:

n_s IS VOLUME DENSITY OF COOPER PAIRS

$$\Rightarrow \vec{J}_s = \frac{e^*\hbar}{m^*}\mathrm{Im}\left\{\sqrt{n_s}e^{-i\theta}\sqrt{n_s}\,i(\vec{\nabla}\theta)e^{i\theta}\right\} - \frac{e^{*2}}{m^*}n_s\vec{A} = \frac{e^*}{m^*}n_s\left[\hbar\vec{\nabla}\theta - e^*\vec{A}\right]$$

$$\Rightarrow \hbar(\vec{\nabla}\theta) = e^*\left(\vec{A} + \Lambda\vec{J}_s\right), \quad \Lambda = \frac{m^*}{(e^*)^2 n_s}.$$

TAKE $\nabla\times$ ON BOTH SIDES:

$$\hbar\underbrace{\nabla\times(\vec{\nabla}\theta)}_{=0} = e^*\left(\underbrace{\vec{\nabla}\times\vec{A}}_{B} + \Lambda\vec{\nabla}\times\vec{J}_s\right) = \vec{0} \Rightarrow \boxed{\vec{B} = -\Lambda\vec{\nabla}\times\vec{J}_s}$$

"2nd LONDON RELATION"

London gauge

$\vec{B} = -\Lambda \vec{\nabla} \times J_s$ IS VALID FOR ALL GAUGE CHOICES.

SPECIAL CHOICE: CHOOSE ELECTRICAL SCALAR POTENTIAL $\Phi = 0$, $\vec{E} = -\vec{\nabla}\Phi - \frac{\partial \vec{A}}{\partial t} = -\frac{\partial \vec{A}}{\partial t}$.

AND CHOOSE

$$\begin{cases} \vec{\nabla}\cdot\vec{A} = -\mu_0 \varepsilon_0 \frac{\partial \Phi}{\partial t} = 0 \\ \hat{n}\cdot\vec{A}\Big|_{\text{SURFACE OF SC}} = -\Lambda \hat{n}\cdot\vec{J}_s \end{cases}$$

FOR $\hat{n}$ NORMAL TO SC SURFACE.

$$\underbrace{\vec{\nabla}\cdot \hbar \vec{\nabla}\theta}_{\hbar D^2\theta} = e^* \left(\underbrace{\vec{\nabla}\cdot\vec{A}}_{=0} + \Lambda \underbrace{\vec{\nabla}\cdot\vec{J}_s}_{= -\frac{\partial n_s}{\partial t} = 0} \right) \Longrightarrow \boxed{D^2\theta = 0}$$

(ASSUME $n_s = \text{CONST}$)

$$\underbrace{\hat{n}\cdot \hbar \vec{\nabla}\theta}_{\hbar D_\perp \theta|_{\text{SURF}}} = e^* \underbrace{\left(\hat{n}\cdot\vec{A} + \hat{n}\cdot\Lambda\vec{J}_s \right)}_{=0} \Longrightarrow D_\perp \theta = 0 \text{ AT SC SURFACE}$$

$$\begin{cases} D^2\theta = 0 \\ D_\perp\theta|_s = 0 \end{cases} \Rightarrow \theta = \text{CONSTANT IN ALL CONTIGUOUS SC MATERIALS.}$$

$\theta = \text{const} \Rightarrow \quad \hbar \underbrace{(\vec{\nabla}\theta)}_{=0} = e^*(\vec{A} + \Lambda \vec{J}_s)$

$$\Rightarrow \boxed{\vec{J}_s = -\frac{1}{\Lambda}\vec{A}}$$

2nd LONDON RELATION IN THE LONDON GAUGE.

(CAREFUL: ONLY VALID FOR SPECIFIC LONDON GAUGE).

▶ TAKE $\frac{\partial}{\partial t}$ ON BOTH SIDES:

$$\frac{\partial}{\partial t}\vec{J}_s = -\frac{1}{\Lambda}\frac{\partial \vec{A}}{\partial t} = \frac{1}{\Lambda}\vec{E}$$

$$\boxed{\frac{\partial \vec{J}_s}{\partial t} = \frac{1}{\Lambda}\vec{E}}$$

1st LONDON RELATION (VALID IN ALL GAUGES!)

Explanation of the Meissner effect, penetration depth

- Plug Maxwell's $J_S = \frac{1}{\mu_0}\nabla \times B$ into 2nd London eqn:

$$\vec{B} = -\Lambda \vec{\nabla}\times\vec{J}_s = -\Lambda\vec{\nabla}\times\left(\frac{1}{\mu_0}\vec{\nabla}\times\vec{B}\right) = -\frac{\Lambda}{\mu_0}\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = -\frac{\Lambda}{\mu}\left[\vec{\nabla}\underbrace{(\vec{\nabla}\cdot\vec{B})}_{=0} - \nabla^2\vec{B}\right]$$

DEFINE $\lambda_L^2 = \frac{\Lambda}{\mu_0} \Rightarrow \boxed{\lambda_L = \sqrt{\frac{m^*}{\mu_0 (e^*)^2 n_s}}}$ "LONDON PENETRATION DEPTH".

$$\Rightarrow \boxed{\nabla^2\vec{B} = \frac{1}{\lambda_L^2}\vec{B}}$$

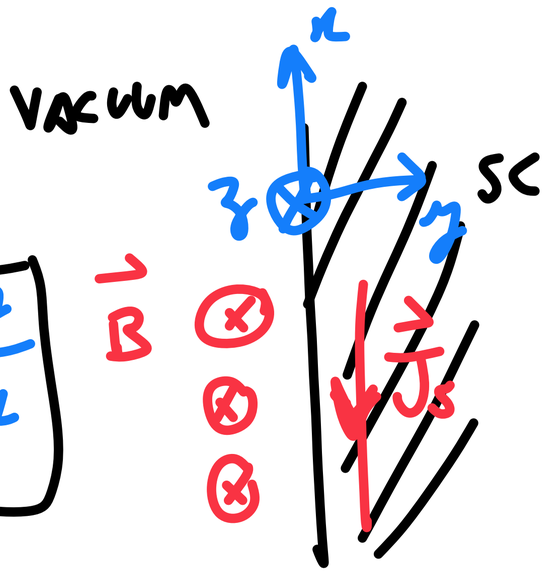
- Solve for a SC half space:

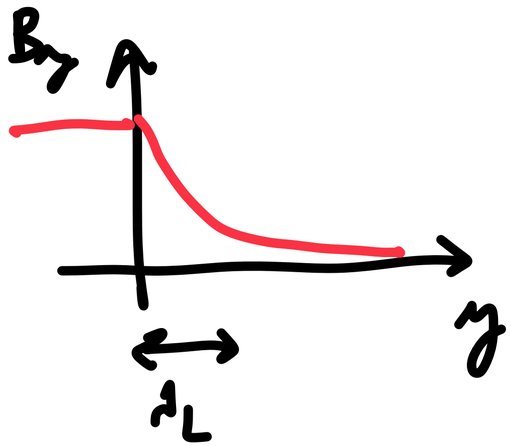
CONSIDER A SC FOR $y \geq 0$

VACUUM FOR $y < 0$

$\vec{B} = B(y)\hat{z}$

$$\Rightarrow \frac{\partial^2 B}{\partial y^2} = \frac{1}{\lambda_L^2}B \Rightarrow \boxed{B(y) = B(0)e^{-\frac{y}{\lambda_L}}}$$





Meissner effect for supercurrent

$$\nabla \times \vec{B} = -\Lambda \nabla \times \left(\vec{\nabla} \times \vec{J}_s \right)$$

$$\Rightarrow \mu_0 \vec{J}_s = -\Lambda \left[\vec{\nabla} \underbrace{\left(\vec{\nabla} \cdot \vec{J}_s \right)}_{= -\frac{\partial n_s}{\partial t} = 0} - \nabla^2 \vec{J}_s \right] = \Lambda \nabla^2 \vec{J}_s$$

FOR n_s = CONST.

$$\Rightarrow \boxed{\nabla^2 \vec{J}_s = \frac{1}{\lambda_L^2} \vec{J}_s}$$

IN A SC, DC CURRENT FLOWS AT THE SURFACE!

SAME FOR AC CURRENTS IN A SC, UP TO $\nu \sim \frac{2\Delta}{h} \sim 100\,\mathrm{GHz}$

Compare to non-SC "ideal" metal with zero resistance

DRUDE MODEL:

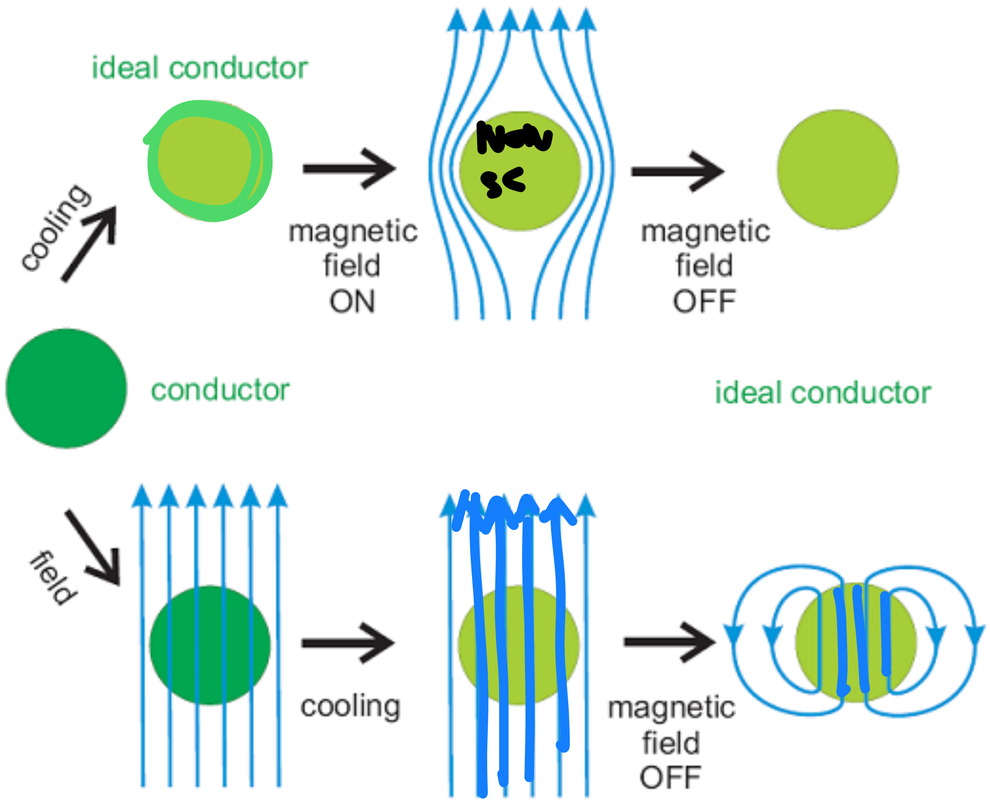
$$\begin{cases} m\dfrac{d\vec{v}}{dt} = e\vec{E} - \underbrace{\dfrac{m}{\tau}\vec{v}} \\ \vec{J} = ne\vec{v} \end{cases}$$

ASSUME $\tau = \infty$ (METAL HAS ZERO RESISTIVITY)

$$\Rightarrow \frac{\partial \vec{J}}{\partial t} = \frac{ne^2}{m}\vec{E}$$

COMPARE TO $\dfrac{\partial \vec{J}_s}{\partial t} = \dfrac{n_s (e^*)^2}{m^*}\vec{E}$

IDENTICAL, EXCEPT FOR e^*, m^*.



$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} \Rightarrow \nabla\times\frac{\partial\vec{B}}{\partial t} = \mu_0\frac{\partial\vec{J}}{\partial t} = \mu_0\frac{ne^2}{m}\vec{E}$$

$$\Rightarrow \underbrace{\nabla\times\nabla\times\frac{\partial\vec{B}}{\partial t}}_{=-\frac{\partial}{\partial t}\nabla^2\vec{B}} = \mu_0\frac{ne^2}{m}\underbrace{\nabla\times\vec{E}}_{=-\frac{\partial\vec{B}}{\partial t}} \Rightarrow \frac{\partial}{\partial t}\left[\nabla^2\vec{B} - \frac{1}{\lambda'^2}\vec{B}\right] = \vec{0} \Rightarrow \vec{B} \sim \vec{B}_0 e^{-\frac{x}{\lambda'}} + \vec{B}_{TRAP}$$

CAN BE NONZERO!

Effective penetration depth λ

- The penetration depth measured in experiments is affected by two additional length scales: "bare" coherence length ξ_0 and electron mean-free path ℓ.

THIS HAPPENS BECAUSE SC ARE "NONLOCAL". BUT LONDON EQNS ASSUME LOCALITY.

NONLOCALITY: $\vec{J}_s(\vec{r},t) = -\frac{3}{4\pi\xi_0\Lambda}\int d^3r' \, \frac{\vec{R}\left[\vec{R}\cdot\vec{A}(\vec{r}',t)\right]e^{-\frac{R}{\xi}}}{R^4}$

PIPPARD EQN

$\vec{R} = (\vec{r}-\vec{r}')$

$\boxed{\text{LONDON IS LOCAL}}$: $\vec{J}_s(\vec{r},t) = -\frac{1}{\Lambda}\vec{A}(\vec{r},t)$

ξ IS AFFECTED BY MEAN-FREE PATH ℓ: $\xi = \frac{1}{\xi_0} + \frac{1}{\ell} \Rightarrow \xi = \frac{\xi_0 \ell}{\xi_0 + \ell} \approx \begin{cases} \ell & \text{WHEN } \ell \ll \xi_0 \text{ "DIRTY LIMIT"} \\ \xi_0 & \text{WHEN } \ell \gg \xi_0 \text{ "CLEAN LIMIT"} \end{cases}$

IMPACT ON λ:

WHEN $\xi \ll \lambda_L \Rightarrow$ LONDON WITH $\lambda \approx \lambda_L\sqrt{\frac{\xi_0}{\xi}}$ INSTEAD OF λ_L.
"LONDON LIMIT" WORKS

WHEN $\xi \gg \lambda_L \Rightarrow$ LONDON DOES NOT WORK, NEED TO ACCOUNT FOR NONLOCALITY.
"PIPPARD LIMIT"

→ BUT LONDON "OK" FOR CALCULATIONS OF MAGNETIC ENERGY PROVIDED YOU USE $\lambda = (\lambda_L^2 \xi)^{1/3}$

Superconductor	λ_L (nm)	ξ_0 (nm)
Al	16	1600
In	19	490
Nb	39	38
Pb	37	83
Sn	35	250

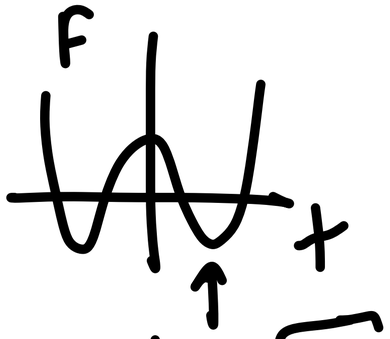
Back to G-L: Critical field

- Neglect fields and gradients inside a bulk SC:

$$\frac{\delta \mathcal{F}}{\delta \Psi^*} = \frac{\partial \mathcal{F}}{\partial \Psi^*} - \nabla \cdot \frac{\partial \mathcal{F}}{\partial \nabla \Psi^*} = \alpha \Psi + \beta |\Psi|^2 \Psi + \underbrace{\frac{1}{2m^*}\left(\frac{\hbar}{i}\nabla - e^* \boldsymbol{A}\right)^2}_{=0} \Psi = 0$$

$$\Rightarrow (\alpha + \beta|\Psi|^2)\Psi = 0 \Rightarrow |\Psi|^2 = n_s = -\frac{\alpha}{\beta}$$

$$= -\frac{\alpha'}{\beta}\left(\frac{T}{T_c} - 1\right) > 0 \text{ WHEN } T < T_c$$



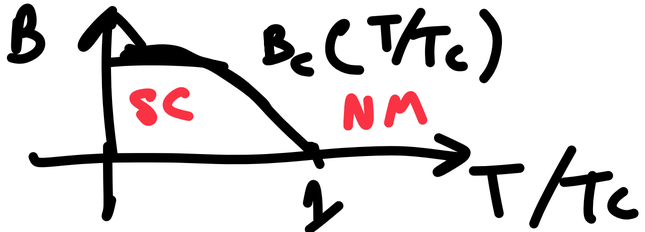
$$\Psi_{eq} = \sqrt{\frac{-\alpha}{\beta}}$$

PLUG $|\Psi|^2$ BACK INTO FREE ENERGY:

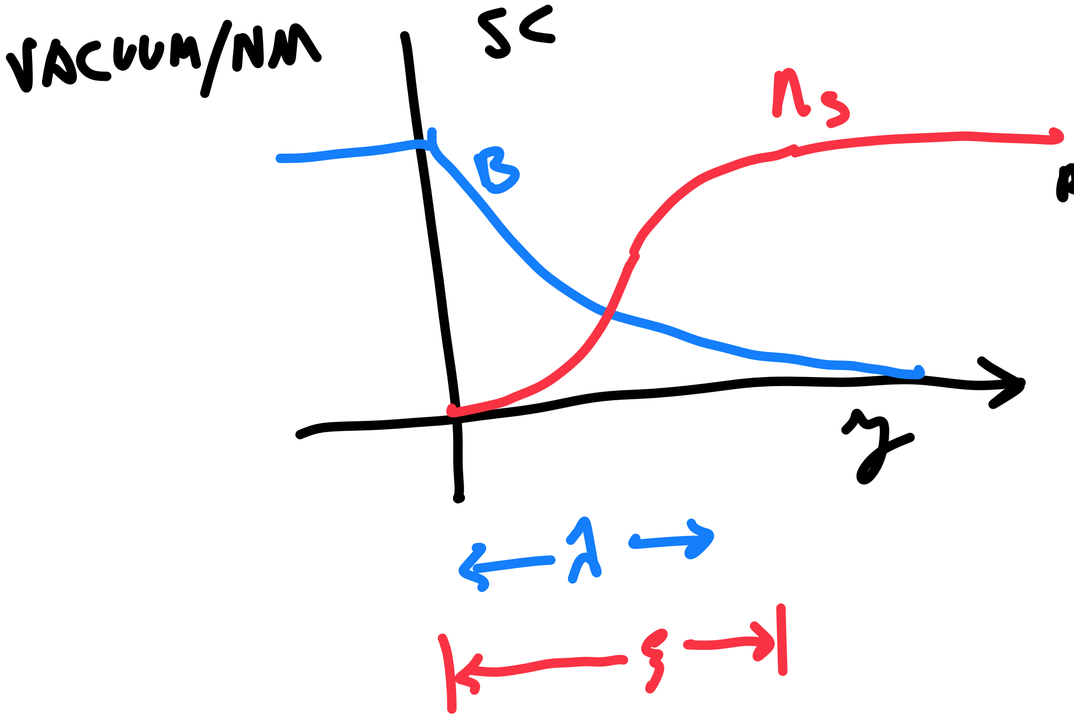
$$F_s = F_N + \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 = F_N + \alpha\left(-\frac{\alpha}{\beta}\right) + \frac{\beta}{2}\left(-\frac{\alpha}{\beta}\right)^2 = F_N \underbrace{- \frac{1}{2}\frac{\alpha^2}{\beta}}_{\text{ENERGY BENEFIT OF BEING SC.}}$$

FROM THERMODYNAMICS: $F_s - F_N = -\frac{1}{2\mu_0} B_c^2$ (SEE de GENNES CHAPTER 1 FOR PROOF!)

$$\Rightarrow \frac{1}{2\mu_0} B_c^2 = \frac{1}{2}\frac{\alpha^2}{\beta} \Rightarrow \boxed{B_c = \sqrt{\frac{\mu_0}{\beta}}\,\alpha' \left|\frac{T}{T_c} - 1\right|}$$

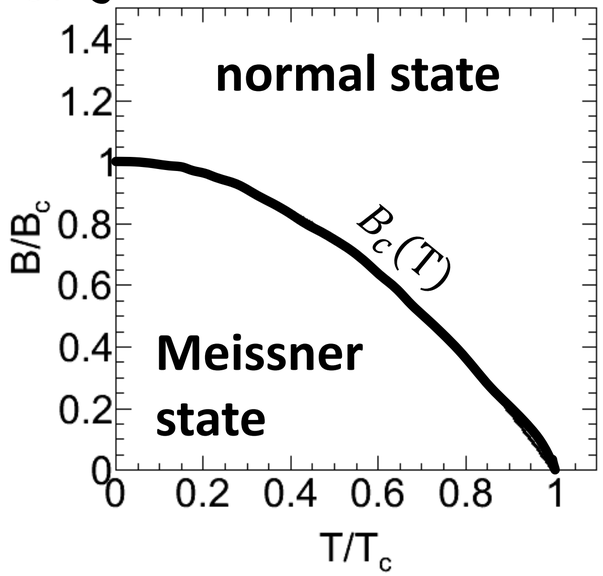


H vs. T phase diagram for SCs: Type I and type II



Type-I

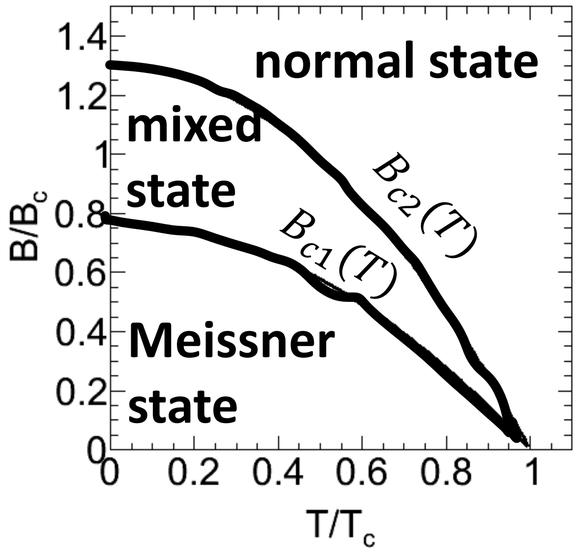
EXACT RESULT G-L: $\kappa = \frac{\lambda}{\xi} < \frac{1}{\sqrt{2}} = 0.71$



$\kappa_{Pb} \sim \frac{28\text{ nm}}{71\text{ nm}} \sim 0.40$

Type-2

$\kappa = \frac{\lambda}{\xi} > \frac{1}{\sqrt{2}} = 0.71$



$\kappa_{Nb} \sim \frac{36\text{ nm}}{39\text{ nm}} \sim 0.92$

ENERGY OF NM/SC DOMAIN WALL:

$$\Delta E \approx \underbrace{\frac{1}{2\mu_0} B_c^2 A \xi}_{} - \underbrace{\frac{1}{2\mu_0} B^2 A \lambda}_{} < 0 \quad \text{WHEN} \quad B > \sqrt{\frac{\xi}{\lambda}}\, B_c \equiv B_{c1}$$

AREA

n_s SUPPRESSED WITHIN ξ OF SURFACE, NO SC ENERGY GAIN

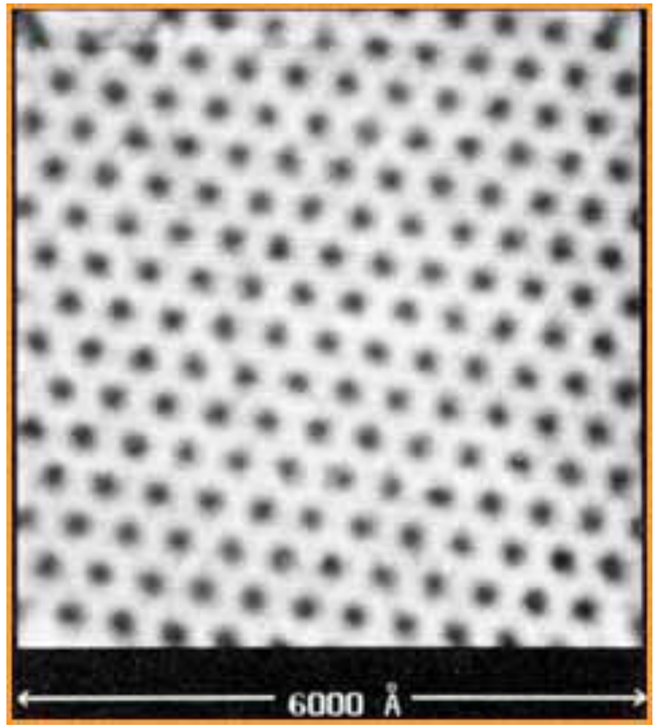
SC CAN ACCEPT B WITHIN λ OF SURFACE, DOES NOT PENALIZE ENERGY.

WHEN $\xi \ll \lambda \Rightarrow$ DOMAIN WALLS FORM AT $B > B_{c1}$.

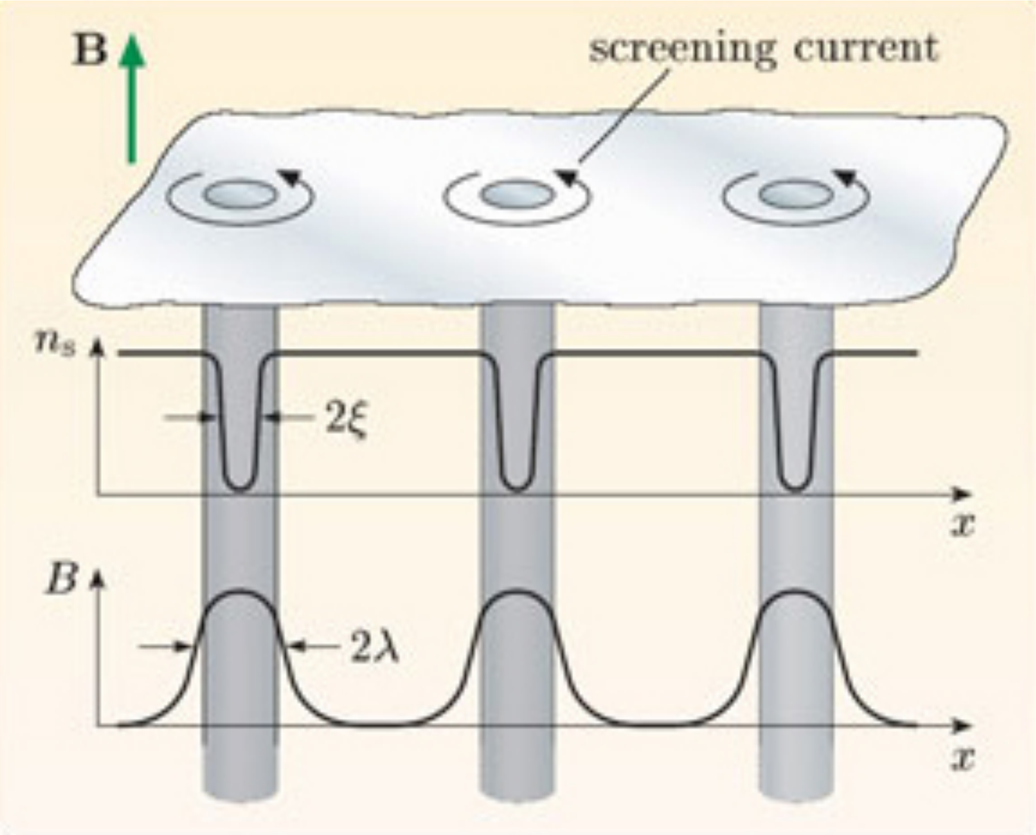
$\Rightarrow$ TYPE II SC!

Mixed state of a type-II SC

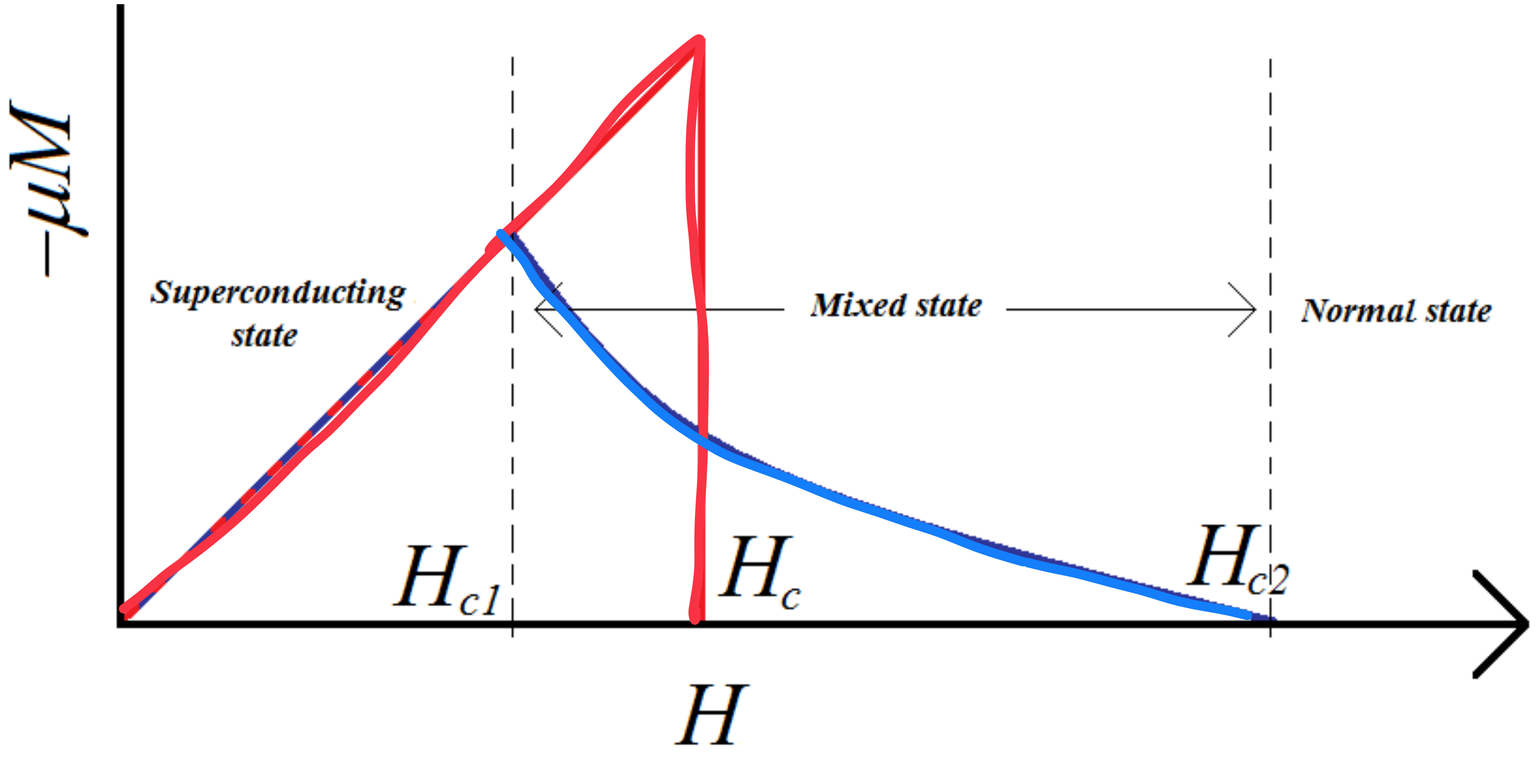
- In Type-II superconductors flux tubes are created each carrying one flux quantum (the minimal flux allowed by quantum mechanics)
- Flux tubes are repulsive creating therefore the vortex lattice



STM image of Vortex lattice, 1989
H. F. Hess et al. Phys. Rev. Lett. 62, 214 (1989)



Magnetization curve



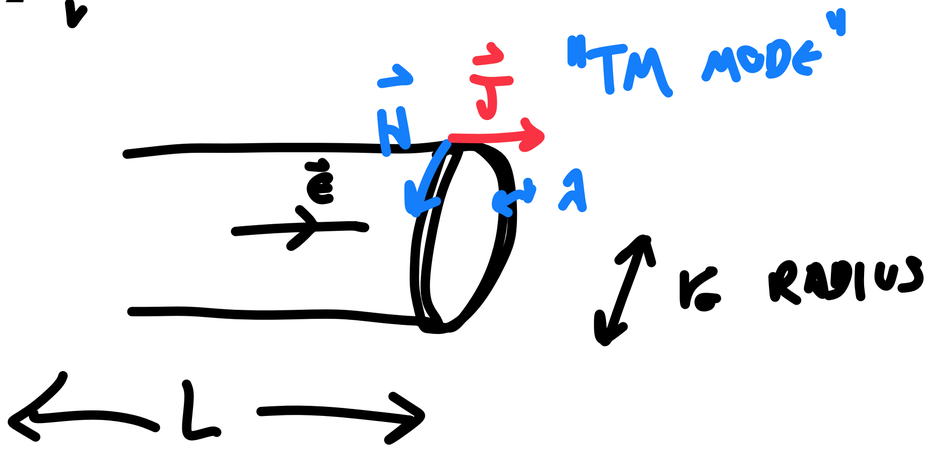
- Under DC fields flux tubes can be pinned – no dissipation
  - SC magnets are operated between H_{c1} and H_{c2}
- Under RF fields flux tubes oscillate – dissipation
  - RF cavities are operated in the Meissner state

Cavity quality factor and surface resistance

- Quality factor due to electromagnetic (Ohmic) loss:

$$Q = \underbrace{\omega_r \tau}_{= \frac{\omega_r}{|\Delta\omega_r|}} = \omega_r \frac{U_{\text{stored}}}{\underbrace{P_{\text{loss}}}_{=\tau}} = \omega_r \frac{\frac{1}{2}\int_V d^3r \mu \left|\boldsymbol{H}(\boldsymbol{r})\right|^2}{\frac{1}{2}\int_V d^3r \boldsymbol{J}(\boldsymbol{r}) \cdot \boldsymbol{E}(\boldsymbol{r})}$$

$\left(\langle \Delta\omega_r \rangle = \frac{1}{\tau}\right)$



$$\int d^3r\, \vec{J}\cdot\vec{E} = \int d^3r \frac{1}{\sigma_N(\vec{r})} \left|\vec{J}(\vec{r})\right|^2$$

$\vec{E} = \frac{1}{\sigma_N}\vec{J}$

$$= \frac{\overbrace{L\, 2\pi r_0}^{A}\, \overbrace{\lambda^2 J^2}^{H^2}}{\sigma_N\, \lambda}$$

FROM $\nabla\times H = J \Rightarrow H \approx \lambda J$

$$= \int d^2r \frac{1}{\sigma_N \lambda} \left|\vec{H}(\vec{r})\right|^2$$

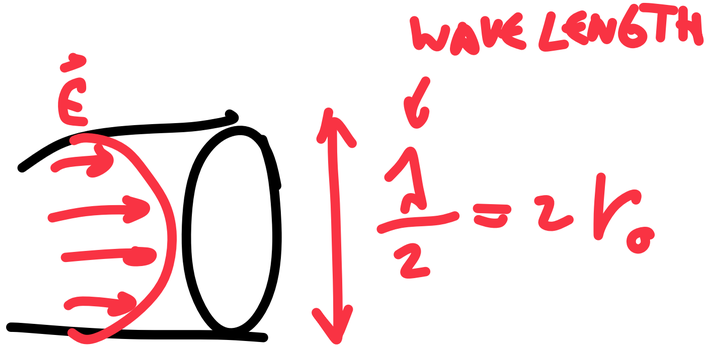
$R_s = \frac{1}{\sigma_N \lambda}$ IS "SURFACE RESISTANCE".

G depends on geometry and is independent of size

$$Q = \omega_r \frac{\int d^3r\, \mu |\vec{H}|^2}{\int_S d^2r\, R_s |\vec{H}|^2} = \frac{1}{R_s} \underbrace{\left(\frac{\omega_r \int_V d^3r\, \mu |\vec{H}|^2}{\int_S d^2r\, |\vec{H}|^2} \right)}_{G} = \frac{G}{R_s}$$

G IS "GEOMETRICAL FACTOR IN OHMS.

ONLY DEPENDS ON GEOMETRY, NOT ON SIZE.



TM MODE:

$$\omega_r = \frac{2\pi c}{4r_0} \propto \frac{1}{r_0}$$

$$G \approx \frac{\omega_r \cancel{\pi} r_0^{\cancel{2}} \cancel{L} \cancel{H^2}}{2\cancel{\pi} \cancel{r_0} \cancel{L} \cancel{H^2}} = \frac{\omega_r r_0}{2} \sim 1$$

<u>G is a geometrical factor</u>

- Elliptical cavity $G \sim 250\ \Omega$
- Spoke cavity $G \sim 133\ \Omega$
- Quarter-wave resonator $G \sim 30\ \Omega$

Origin of surface resistance: Resistivity due to quasiparticles

- NORMAL METAL (Cu): $R_s = \frac{1}{\sigma_N \delta_{\text{SKIN DEPTH}}} = \frac{1}{\sigma_N} \frac{1}{\sqrt{\frac{2}{\omega_r \mu \sigma_N}}} = \sqrt{\frac{\omega_r \mu}{2\sigma_N}} \propto \sqrt{\frac{f_r}{\sigma_N}}$.

- SC: 1ST LONDON: $\frac{\partial \vec{J}_s}{\partial t} = \frac{1}{\Lambda}\vec{E} \xrightarrow{\text{FOURIER}} -i\omega \vec{J}_s(\omega) = \frac{1}{\Lambda}\vec{E}(\omega) \Rightarrow \vec{J}_s(\omega) = \underbrace{\frac{i}{\mu_0 \lambda^2 \omega}}_{=i\sigma_2}\vec{E}(\omega)$

OPTICAL CONDUCTIVITY OF SC IS IMAGINARY!

$$\vec{J}_s(\omega) = (\sigma_1 + i\sigma_2)\vec{E}(\omega)$$

REAL PART DUE TO THERMAL QUASIPARTICLES.

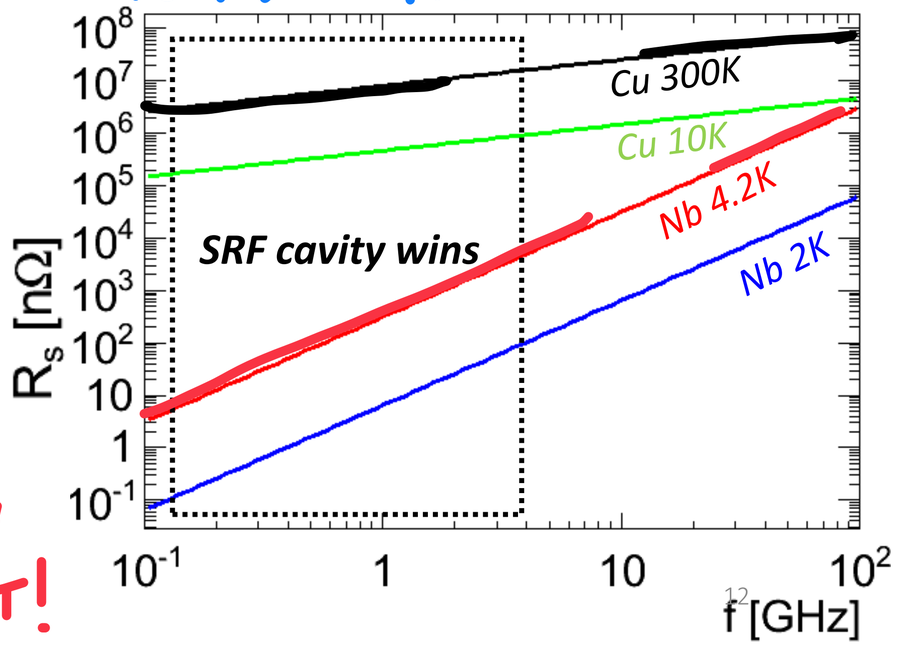
$$R_s = \mathrm{Re}\left\{\frac{1}{(\sigma_1 + i\sigma_2)\lambda}\right\} = \mathrm{Re}\left\{\frac{\sigma_1 - i\sigma_2}{(\sigma_1^2+\sigma_2^2)\lambda}\right\} \approx \frac{\sigma_1}{\sigma_2^2 \lambda}$$

$\sigma_2 \gg \sigma_1$ FOR SC

PLUG $\sigma_2 = \frac{1}{\mu_0 \lambda^2 \omega}$:

$$R_s = \sigma_1 \mu_0^2 \lambda^3 \omega_r^2 \propto \boxed{\sigma_1 \lambda^3 f_r^2}$$

COMPARE TO NM, TOTALLY DIFFERENT!



A. Myiazaki, "Basics of RF superconductivity and Nb material", SRF 2023

R_S vs mean-free path l

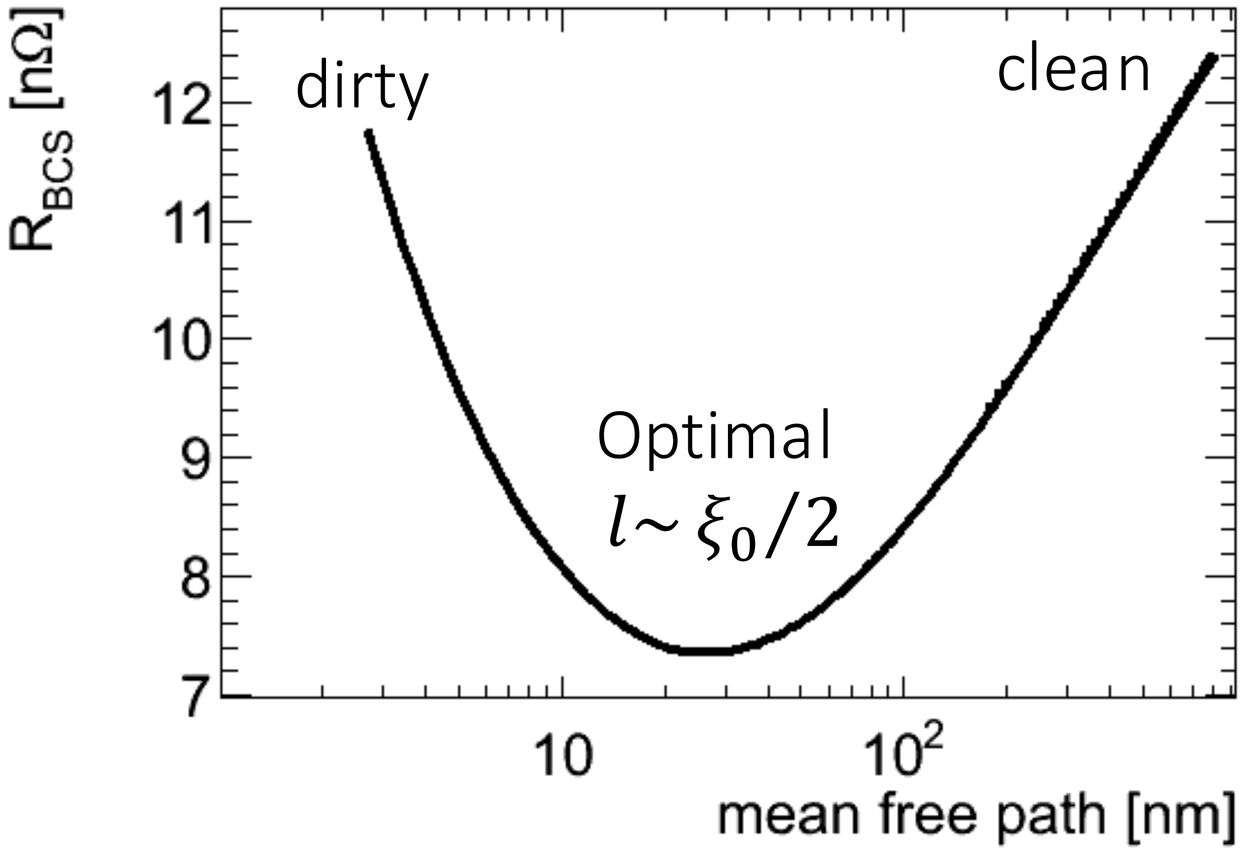
$$R_s = \sigma_1 \mu_0^2 \lambda^3 \omega_r^2$$

$$\sigma_1 \propto l \qquad \lambda \propto \lambda_L \sqrt{\frac{\xi_0}{l}} \propto \sqrt{1+\frac{\xi_0}{l}}$$

$$R_s \propto l\left(1+\frac{\xi_0}{l}\right)^{3/2} = \left(l^{2/3} + \xi_0 l^{-1/3}\right)^{3/2}$$

$$\frac{dR_s}{dl} \propto \frac{2}{3} l^{-1/3} - \frac{1}{3}\xi_0 l^{-4/3} = 0$$

$$\Rightarrow \boxed{l = \frac{\xi_0}{2}}$$



Counter intuitively, super clean material is not ideal for SRF cavities!

→ Heat treatment, doping, etc to make ***surface*** dirty

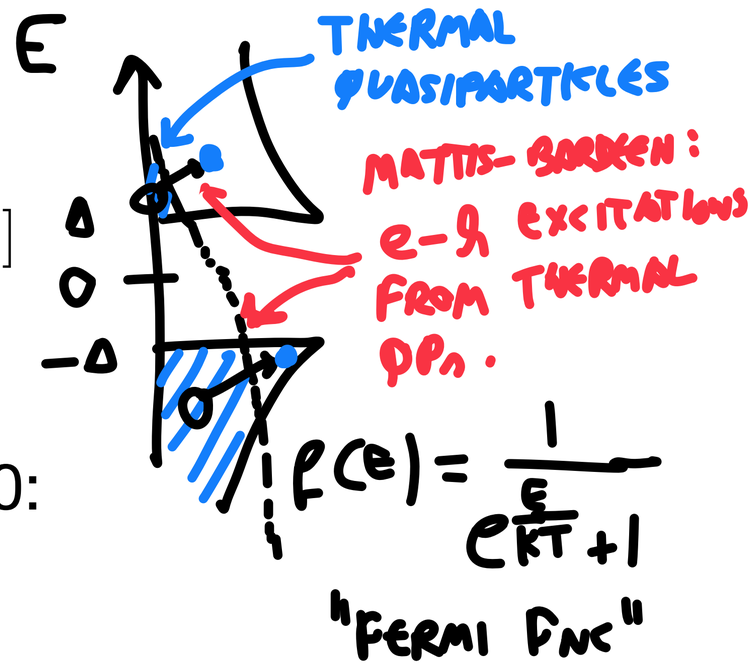
A. Myiazaki, "Basics of RF superconductivity and Nb material" , SRF 2023 @Michigan State University

Optical conductivity from BCS theory

D.C. Mattis and J. Bardeen, Phys. Rev. **111**, 412 (1958)

- For $\hbar\omega < 2\Delta$, thermal quasiparticles cause $\sigma_1>0$ (real part of conductivity):

$$\frac{\sigma_1}{\sigma_N} = \frac{2}{\hbar\omega}\int_{\Delta}^{\infty} dE \frac{E(E+\hbar\omega)+\Delta^2}{\sqrt{E^2-\Delta^2}\sqrt{(E+\hbar\omega)^2-\Delta^2}}\left[f(E)-f(E+\hbar\omega)\right]$$



- BCS also predicts deviations from the London σ_2 when T>0:

$$\frac{\sigma_2}{\sigma_N} = \frac{1}{\hbar\omega}\int_{\Delta-\hbar\omega}^{\Delta} dE \frac{E(E+\hbar\omega)+\Delta^2}{\sqrt{\Delta^2-E^2}\sqrt{(E+\hbar\omega)^2-\Delta^2}}\left[1-2f(E+\hbar\omega)\right]$$

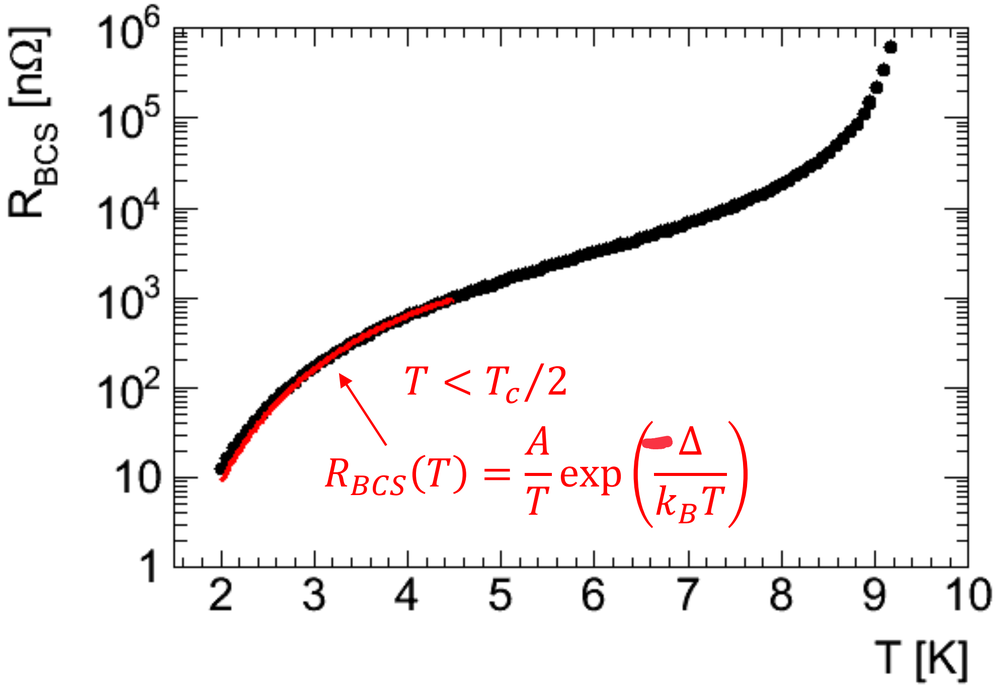
Microscopic calculation of R_S using BCS

- Calculate $R_S = \dfrac{\sigma_1}{\sigma_2^2 \lambda}$ using Mattis-Bardeen and two-fluid expression for λ(T):

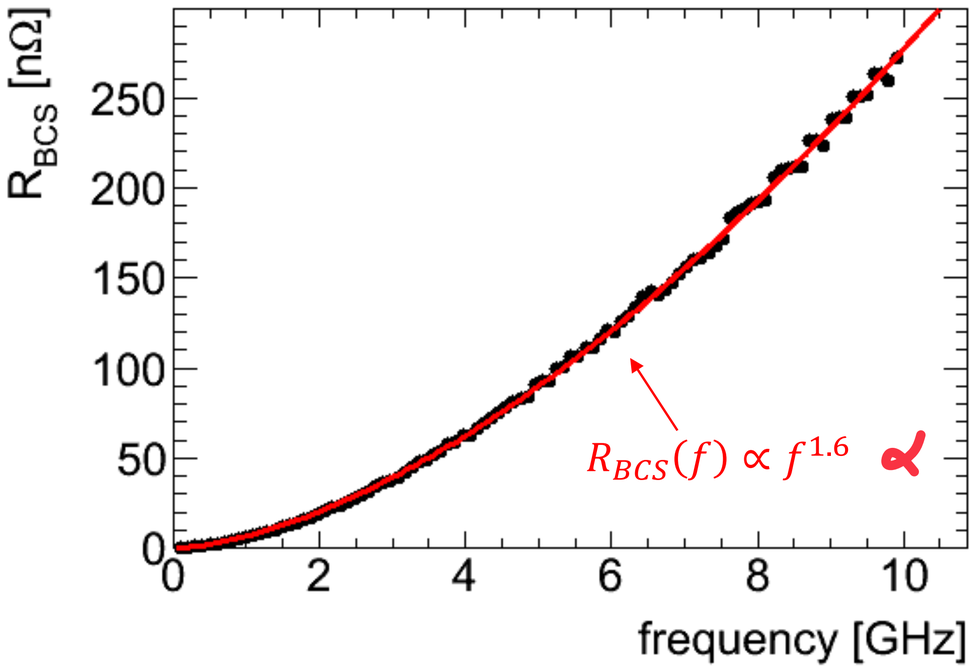
$$\lambda(T) = \frac{\lambda(0)}{\sqrt{1-(T/T_c)^4}}$$

← EXCELLENT APPROXIMATION FOR λ(T) IN ALL CASES.

Temperature dependence is exponential



Frequency dependence between $f^{1.5}$ and f^2



$\propto f^{\gamma}$ WITH $\gamma = 1.5 - 2$

References and further reading

- Lecture notes from A. Myiazaki, “Basics of RF superconductivity and Nb material” , SRF 2023 @Michigan State University, download from: https://srf2023.vrws.de/talks/thtut100_talk.pdf

IF YOU DO PARTICLE PHYSICS, READ THIS:

- S. Weinberg, Superconductivity for particular theorists, Prog. Theor. Phys. Supplement **86**, 43 (1986).
- M. Tinkham, *Introduction to Superconductivity*, 2nd ed. (McGraw-Hill, 1996).
- T. Van Duzer and C. W. Turner, *Superconductive Devices and Circuits*, 2nd ed. (Prentice-Hall, 1999).
- P.G. de Gennes, Superconductivity of metals and alloys (Westview press, 1999).
- N. Gorgichuk, T. Junginger, and R. de Sousa, Phys. Rev. Appl. **19**, 024006 (2023).
- P. S. Yapa, T. Makaro, and R. de Sousa, *Impact of nonlocal electrodynamics on flux noise and inductance of superconducting wires*, Phys. Rev. Appl. **11**, 024041 (2019).