Basic crab cavities

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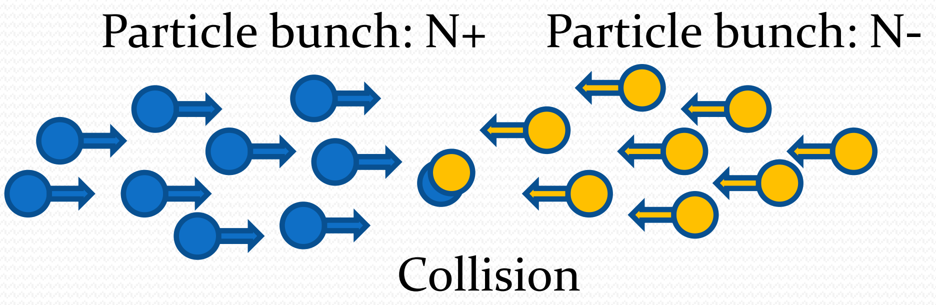
Basic crab cavities

- Luminosity of the collider accelerators and related topics
- RF cavities and parameters
- Pill box type crab cavities, mainly developed for KEKB
- Compact crab cavities, mainly developed for LHC

Colliding beams

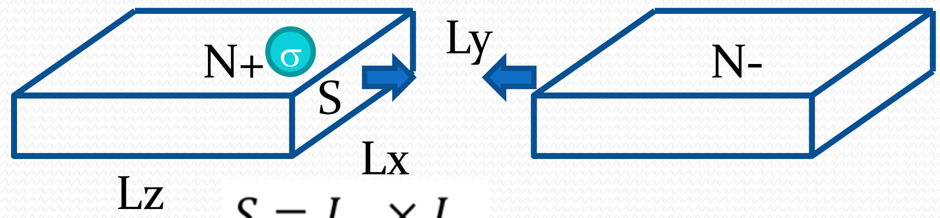
- We consider the luminosity of collider accelerators
- Luminosity is one of the most important parameters
- Number of physics interactions (events) per second = L x σ
  - L: Luminosity
  - σ: Cross section
- L has to be increased to obtain high event rate



$$\frac{dN_{ev}}{dt} = L \cdot \sigma$$

Luminosity

- Calculate luminosity
  - Head on collision
  - Flat beams with uniform distribution
- # of events for a particle with a cross section of σ
- # of events for N+ particles
- If those collisions occur f times per second
- Luminosity for flat beams



$$S = L_x \times L_y$$

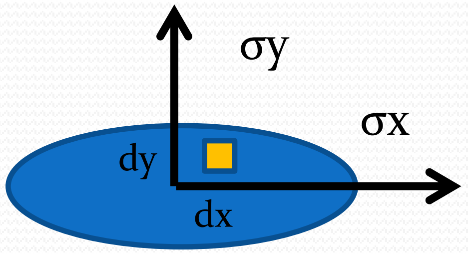
$$N_{ev} = N_{-}\sigma/S$$

$$N_{ev} = N_{+}N_{-}\sigma/S$$

$$\frac{dN_{ev}}{dt} = fN_{+}N_{-}\sigma/S$$

$$L = \frac{N_{+}N_{-}}{L_x L_y} f$$

Luminosity



- In case of Gaussian distribution we have to modify the previous equation
  - Same σx, σy for both beams

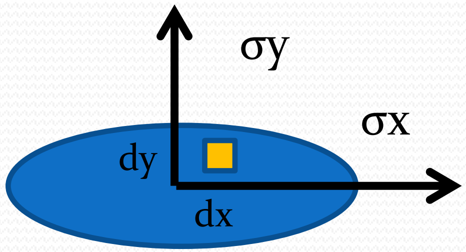
$$L = \frac{N_+ N_-}{L_x L_y} f \qquad \begin{array}{l} L_x \to dx \\ L_y \to dy \end{array}$$

$$N_+ \to N_+ \frac{1}{\sqrt{2\pi}\sigma_{x+}} e^{-\frac{x^2}{2\sigma_{x+}^2}} \frac{1}{\sqrt{2\pi}\sigma_{y+}} e^{-\frac{y^2}{2\sigma_{y+}^2}} dx\, dy$$

$$N_- \to N_- \frac{1}{\sqrt{2\pi}\sigma_{x-}} e^{-\frac{x^2}{2\sigma_{x-}^2}} \frac{1}{\sqrt{2\pi}\sigma_{y-}} e^{-\frac{y^2}{2\sigma_{y-}^2}} dx\, dy$$

$$L = \frac{N_+ N_-}{4\pi \sigma_x \sigma_y} f$$

Luminosity



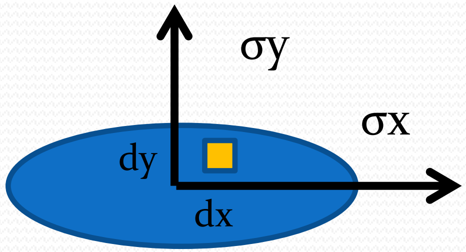
- In case of Gaussian distribution
  - Same σx, σy
  - But the other beam has offset Δx
- Reduction factor

$$L = \frac{N_+ N_-}{4\pi\sigma_x\sigma_y} f$$

$$N_- \to N_- \frac{1}{\sqrt{2\pi}\sigma_{x-}} e^{-\frac{(x-\Delta x)^2}{2\sigma_{x-}^2}} \frac{1}{\sqrt{2\pi}\sigma_{y-}} e^{-\frac{y^2}{2\sigma_{y-}^2}} dx\, dy$$

$$R_{\Delta x} = e^{-\frac{\Delta x^2}{4\sigma_x^2}}$$

Luminosity



- In case of Gaussian distribution
  - Different σx, σy
  - Define Σx and Σy

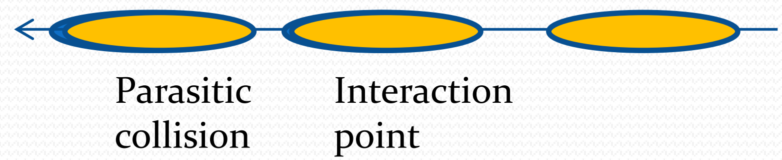
$$\Sigma_x = \sqrt{\sigma_{x+}^2 + \sigma_{x-}^2}$$

$$\Sigma_y = \sqrt{\sigma_{y+}^2 + \sigma_{y-}^2}$$

$$L = \frac{N_+ N_-}{2\pi \Sigma_x \Sigma_y} f$$

Finite angle crossing

- To increase luminosity
  - Increase beam bunches
  - When bunches increased parasitic collisions occur
- To avoid parasitic collisions
  - Beam crossing with finite angle: θc
  - Make beam separation simple
  - Reduction of luminosity
    - Geometrical reduction factor R_L
  - May introduce coupling between synchrotorn and betatron oscillations in circular colliders



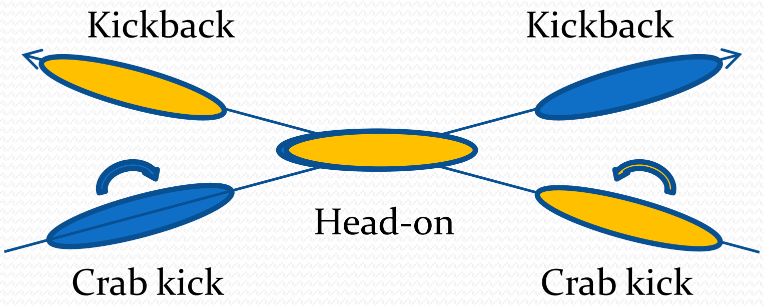
$$R_L = \frac{1}{\sqrt{1+\phi_P^2}}$$

$$\phi_P = \frac{\sigma_z}{\sigma_x} \tan \theta_c / 2$$

Piwinski angle

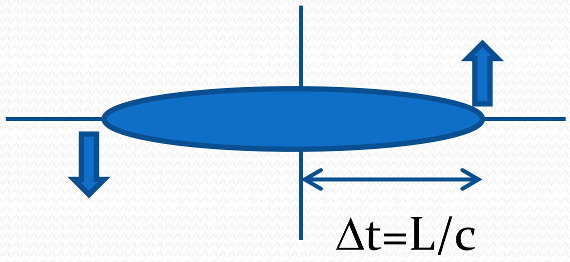
Crab crossing

- Bunch rotation at the IP
  - Horizontal kick at the bunch head
  - Horizontal kick at the bunch tail but opposite direction
  - No kick at the bunch center
- Bunch rotates so that it collides head-on at IP
  - Crab crossing
- Crab cavity kick the bunch with a phase advance of $\pi/2$
- Another crab cavity kicks back the bunch after IP



Required kick voltage

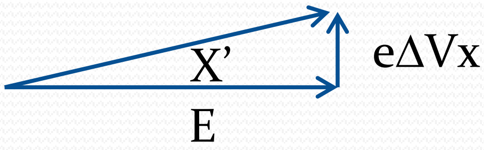
- Kick voltage given to the bunch head
  - L: length from the bunch center
  - c: speed of light
  - ΔV: kick voltage
  - No kick to the bunch center
  - Same kick voltage to the bunch tail, but opposite direction



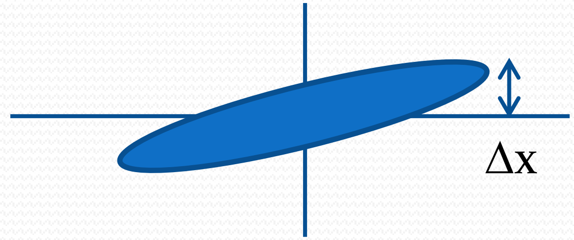
$$\Delta V_x = V_x \sin \omega_{RF} \Delta t \sim V_x \omega_{RF} \frac{L}{c}$$

Required kick voltage

- Kick angle, x'
- Offset at the IP
    - m12: transfer matrix component of the betatron oscillation
    - Δϕ: phase advance (sinΔϕ=1)
- Bunch rotation angle at the IP
    - Half crossing angle
- Required kick voltage



$$x' = \frac{e\Delta V_x}{E} = \frac{\Delta V_x}{E/e}$$



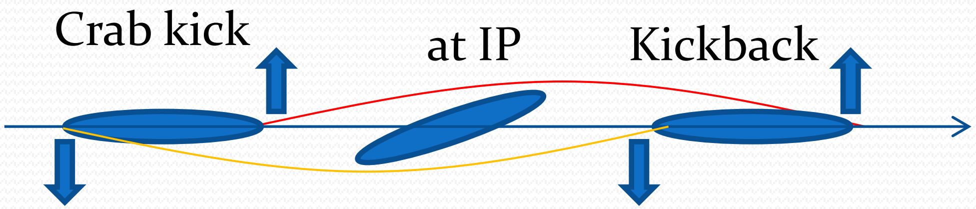
$$\Delta x = m_{12} x' = \sqrt{\beta_{crab}\beta_{IP}} \sin\Delta\phi \, x'$$

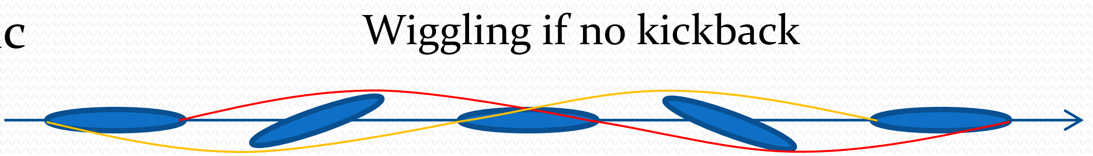
$$\tan\theta = \frac{\Delta x}{L} = \sqrt{\beta_{crab}\beta_{IP}} \frac{V_x \omega_{RF}}{cE/e}$$

$$V_x = \frac{cE/e}{\sqrt{\beta_{crab}\beta_{IP}}\,\omega_{RF}} \theta$$

Single crab cavity scheme

- One crab cavity in the ring
- The beam wiggles in the whole ring
- Cavity location can be chosen any where in the ring
  - near the existing cryogenic facility
- KEKB applied this scheme
  - Two crab cavities fabricated and installed in the HER and LER ring
  - The location is 1 km away from the IP, where a cryogenic facility was operating for the SRF accelerating cavities
  - Great cost reduction





Required kick voltage in single crab scheme

Bunch head

Crab kick $\Delta x' = \frac{V\omega_{RF}}{cE/e} L$ IP

$$\begin{pmatrix} x_3 \\ x_3' \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 = x_3 \\ x_1' = x_3' + \Delta x' \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_1' \end{pmatrix} \Rightarrow M_{(s2,s1)} \Rightarrow \begin{pmatrix} x_2 \\ x_2' \end{pmatrix}$$

$$\Leftarrow M_{(s1+C,s2)} \Leftarrow$$

Wiggling in the ring

Coordinate transformation

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = M_{(s2,s1)} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$

$$\begin{pmatrix} x_3 \\ x_3' \end{pmatrix} = M_{(s1+C,s2)} \begin{pmatrix} x_2 \\ x_2' \end{pmatrix}$$

Transfer matrix

$$M_{(s2,s1)} = \begin{pmatrix} \sqrt{\frac{\beta_2}{\beta_1}}(\cos\psi_{21} + \alpha_1 \sin\psi_{21}) & \sqrt{\beta_2\beta_1}\sin\psi_{21} \\ \frac{1}{\sqrt{\beta_2\beta_1}}(-(\alpha_2 - \alpha_1)\cos\psi_{21} - (1 + \alpha_2\alpha_1)\sin\psi_{21}) & \sqrt{\frac{\beta_2}{\beta_1}}(\cos\psi_{21} - \alpha_1 \sin\psi_{21}) \end{pmatrix}$$

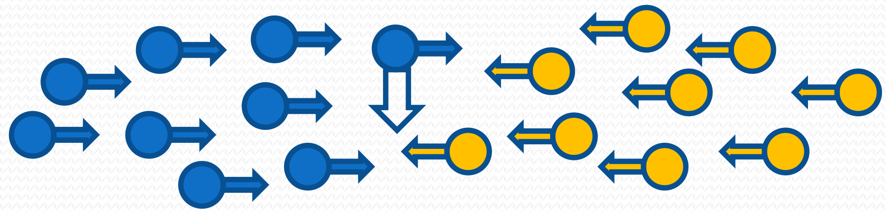
Bunch rotation angle at IP

$$\tan\frac{\theta_c}{2} = \sqrt{\beta_2\beta_1}\,\frac{\cos\left(\frac{\psi_c}{2} - \psi_{21}\right)}{2\sin\frac{\psi_c}{2}}\,\frac{V\omega_{RF}}{cE/e}$$

Phase advance in the ring: $\psi c \sim \pi$

Phase advance at the crab: $\psi 21 = \pi/2$

Beam-beam parameter

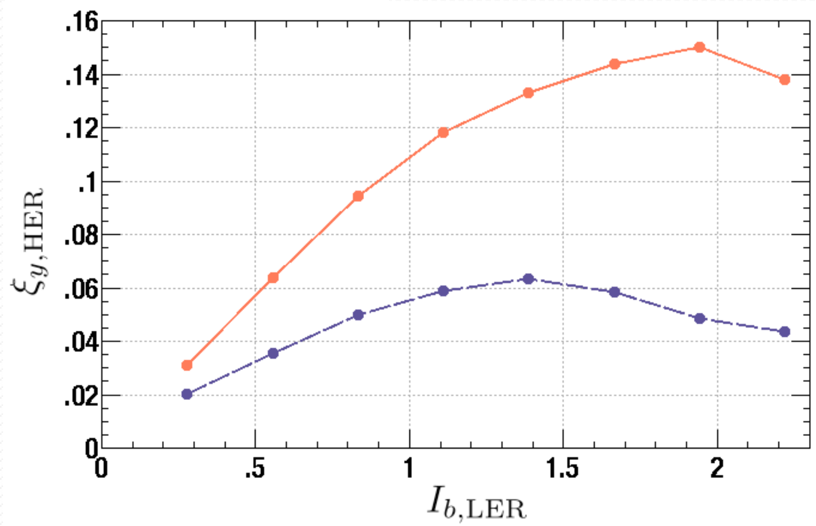


- Beam-beam kick
  - Electromagnetic force of electrons kicks positrons
  - Kicking force is approximately proportional to the distance from the center
- Tune shift
  - This force is like a quadrupole magnet
  - Tune shifts due to this kick
    - Δμ: ξ parameter
    - Beam-beam parameter
- Beam-beam limit
  - If we increase N, σ increases
  - Beam-beam parameter has a limit
    - Bean-bean limit
  - Luminosity has a limit
- Beam-beam simulation predicted ξ could be increased more than twice in the crab crossing

$$\xi_{x,y\pm} = \frac{r_e N_\mp}{2\pi\gamma_\pm} \frac{\beta_{x,y\pm}}{\sigma_{x,y\mp}\sigma_{y\mp}}$$

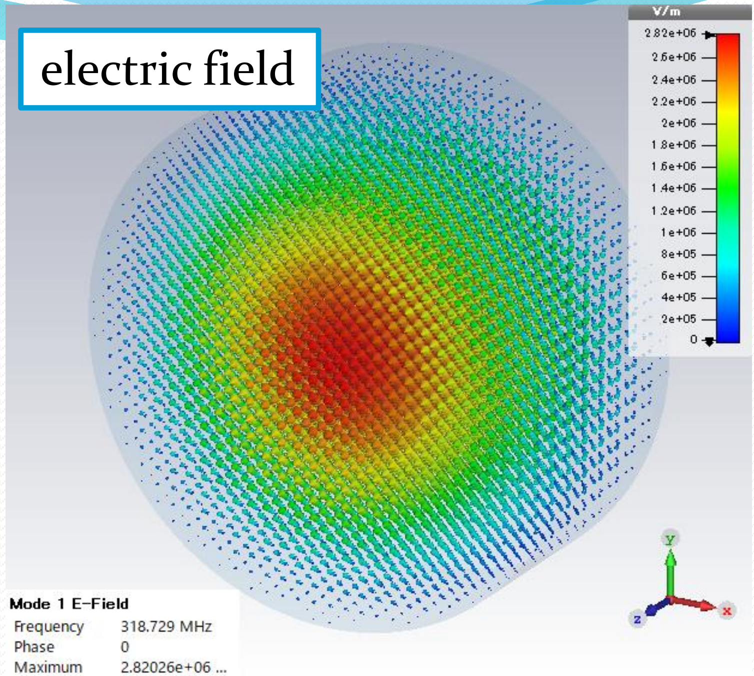
$$L = \frac{N_\pm}{2r_e} f \gamma_\pm \frac{\xi_{y\pm}}{\beta_{y\pm}}$$

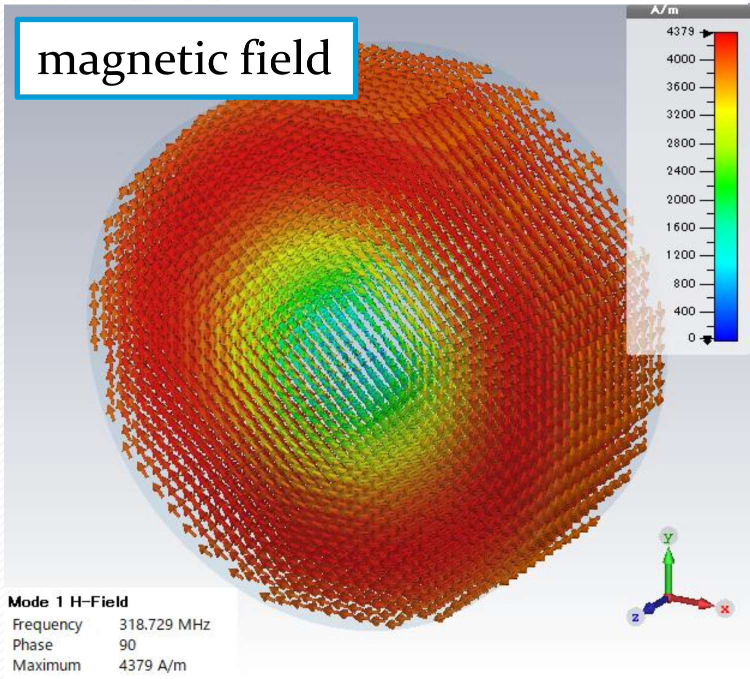
r_e: Classical electron radius
γ: Lorentz factor



RF cavity

- Particle acceleration longitudinally or transversely
- Time varying electric and magnetic fields in a cavity
  - Resonate with the resonant frequency
  - Create large electric and magnetic fields
  - Those fields accelerate charged particles





RF cavity

- Resonant frequency and electro magnetic fields can be obtained from;
  - Maxell's equations
  - Boundary conditions
- Electro-magnetic fields decay in time due to the heating loss on the boundary
  - The ratio of the loss in one cycle Pc to stored energy U characterizes cavity performance
  - Quality factor: Q=ωU/Pc

$$DivB = 0$$

$$DivE = 0$$

$$RotE = -\frac{\partial B}{\partial t}$$

$$RotH = \frac{\partial D}{\partial t}$$

$$Q = \frac{\omega U}{P_c}$$

Beam acceleration

- Lorentz force
  - Charged particle with speed of light enters into a cavity
  - Charged particles obtain acceleration: Lorentz force
- We obtain the accelerating voltage when the cavity has longitudinal electric field
- Accelerating voltage Vc
  - The EM fields vary in time
  - We have to take the travelling time of a particle into account
  - Transit time factor: T
    - The ratio of Vc to the maximum voltage

$$F = e(E + v \times B)$$

$$F_z = eE_z$$

$$V_c = \int_0^d E_z e^{i\omega t} dz = \int_0^d E_z e^{ikz} dz$$

$$V_c = \left| \int_0^d E_z e^{ikz} dz \right|$$

$$T = \frac{\left| \int_0^d E_z e^{ikz} dz \right|}{\left| \int_0^d E_z dz \right|}$$

Shunt impedance

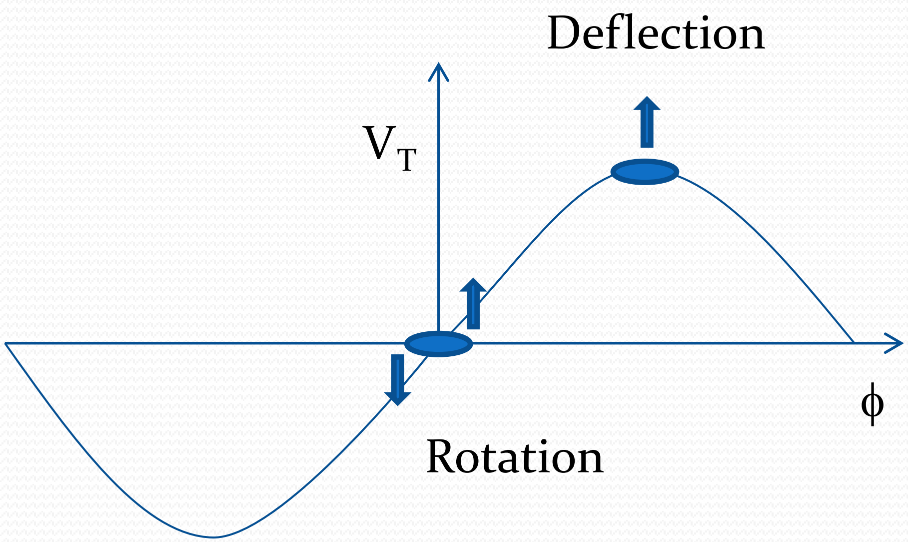
- Shunt impedance
  - Important parameter
  - Ratio of Vc^2 to Pc
  - Efficiency of acceleration
  - In the lumped circuit theory; $R=Vc^2/Pc/2$
- R/Q
  - Another important parameter

$$R = \frac{V_c^2}{P_c}$$

$$R/Q = \frac{V_c^2}{\omega U}$$

Direction of acceleration

- Longitudinal acceleration
  - Beam acceleration
- Transverse acceleration
  - Beam deflection
    - Whole bunch deflects
  - Bunch rotation
    - Bunch head deflects
    - Bunch tail deflects, but in the opposite direction
    - Bunch center does not deflect
    - Crabbing



How to kick the beam

- Lorentz force
  - Particle with v=c along z-axis
- The force accelerates particles
  - Z-comp: acceleration
  - X-comp: horizontal kick
  - Y-comp: vertical kick
- Horizontal kick voltage

$$F = e(E + v \times B)$$

$$v = ce_z = (0, 0, c)$$

$$F_x = e(E_x - cB_y)$$
$$F_y = e(E_x + cB_x)$$
$$F_z = eE_z$$

$$V_x = \int_0^d (E_x - cB_y) e^{ikz}\, dz$$

Panofsky-Wenzel theorem

- Maxwell's eq: $RotE = -\frac{\partial B}{\partial t} = -i\omega B$ ➡ $B_y = \pi \frac{i}{\omega}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)$
- V_x can be given by E only

$$V_x = \int_0^d \left(E_x - i\frac{c}{\omega}\frac{\partial E_x}{\partial z} + i\frac{c}{\omega}\frac{\partial E_z}{\partial x}\right) e^{ikz}\, dz$$

- Note that the first 2 terms are combined
  - Its integral vanish at the boundary because Ex=0

$$(E_x - i\frac{c}{\omega}\frac{\partial E_x}{\partial z})e^{ikz} = -i\frac{c}{\omega}\frac{d}{dz}(E_x e^{ikz})$$

- V_x is given by the gradient of V_z
  - The TE mode does not kick the particle because Ez=0

$$V_x = \int_0^d i\frac{c}{\omega}\frac{\partial E_z}{\partial x} e^{ikz}\, dz = \frac{i}{k}\frac{\partial V_z}{\partial x}$$

Kick voltage

- Kick voltage
  - Horizontal kick
  - Vertical kick
- Shunt impedance
  - T stands for x or y
- R/Q

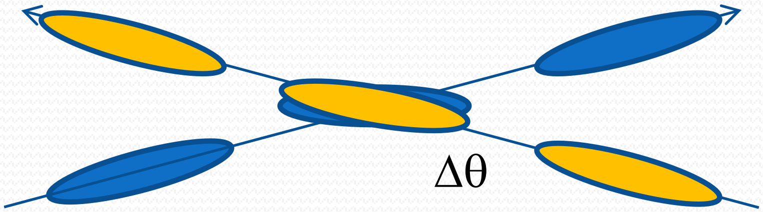
$$V_x = \frac{i}{k}\frac{\partial V_z}{\partial x} \qquad V_y = \frac{i}{k}\frac{\partial V_z}{\partial y}$$

$$R_T = \frac{V_T^2}{P_c}$$

$$R/Q = \frac{V_T^2}{\omega U}$$

Voltage stability

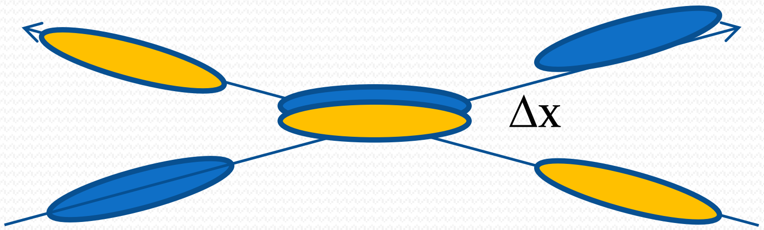
- Voltage error: ΔV
- Bunch rotation angle error: Δθ
  - Proportional to V
- Geometrical reduction factor



$$\Delta\theta = \sqrt{\beta_{crab}\beta_{IP}}\,\frac{\omega_{RF}}{cE/e}\Delta V$$

$$R_L = \frac{1}{\sqrt{1+\phi_P^2}}$$

Phase stability



- Phase error of one crab cavity: $\Delta\phi$
- Transverse kick of the bunch center
  - Proportional to $\Delta\phi$
- Bunch position offset at IP: Δx
- Geometrical reduction factor
- Absolute phase error of two crab cavities has no geometrical reduction
  - Both beams have the same offset at IP
  - Because $\Delta x=0$

$$\Delta x = \sqrt{\beta_{crab}\beta_{IP}}\,\frac{V}{\mathrm{E/e}}\Delta\phi$$

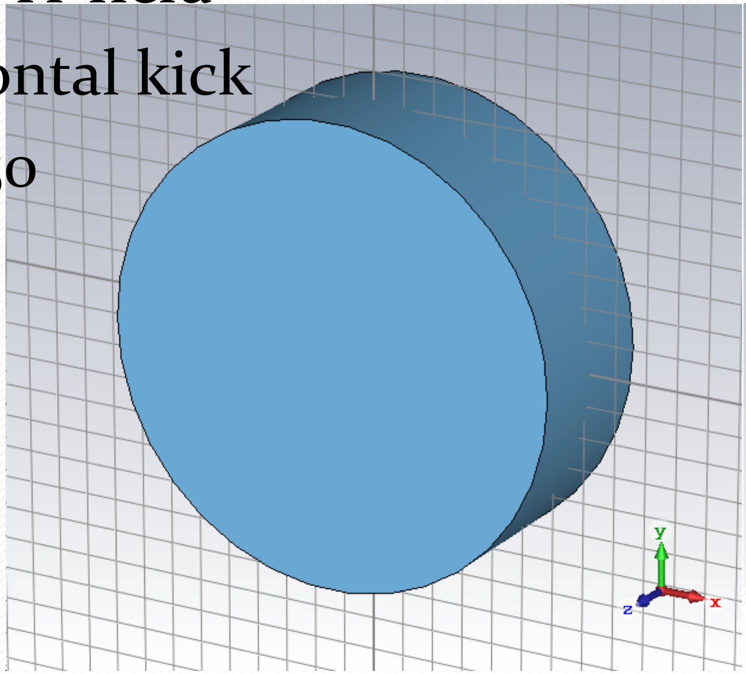
$$R_{\Delta x} = e^{-\frac{\Delta x^2}{4\sigma_x^2}}$$

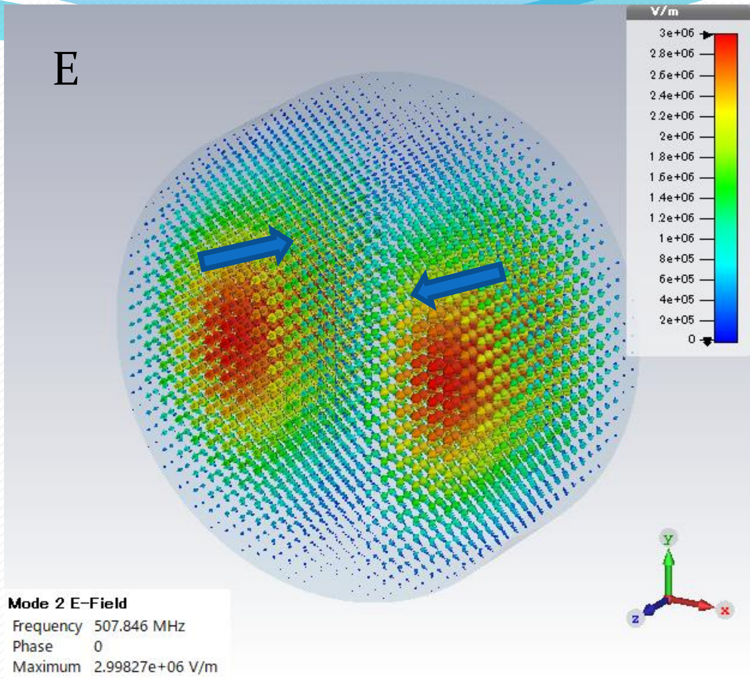
Cavity type

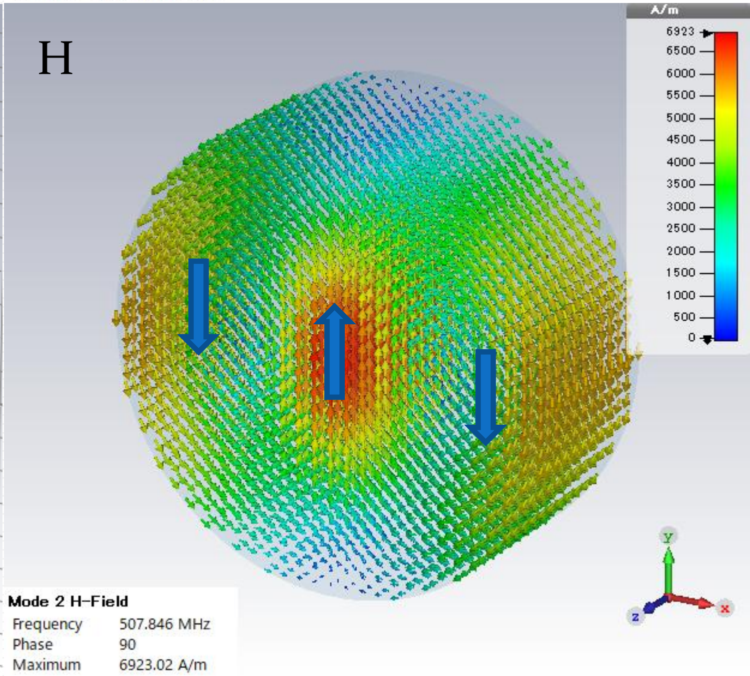
- Pill box
- Rectangular cavity
- Coaxial cavity

Crab cavity shape

- Pill box type
- TM110 dipole mode
  - 500MHz
  - E- and H-field
  - Horizontal kick
  - R/Q~50

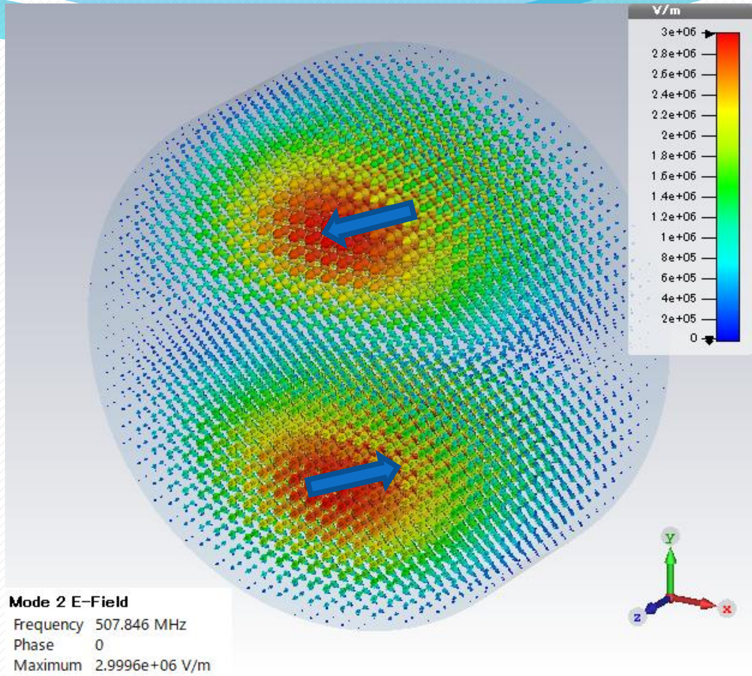


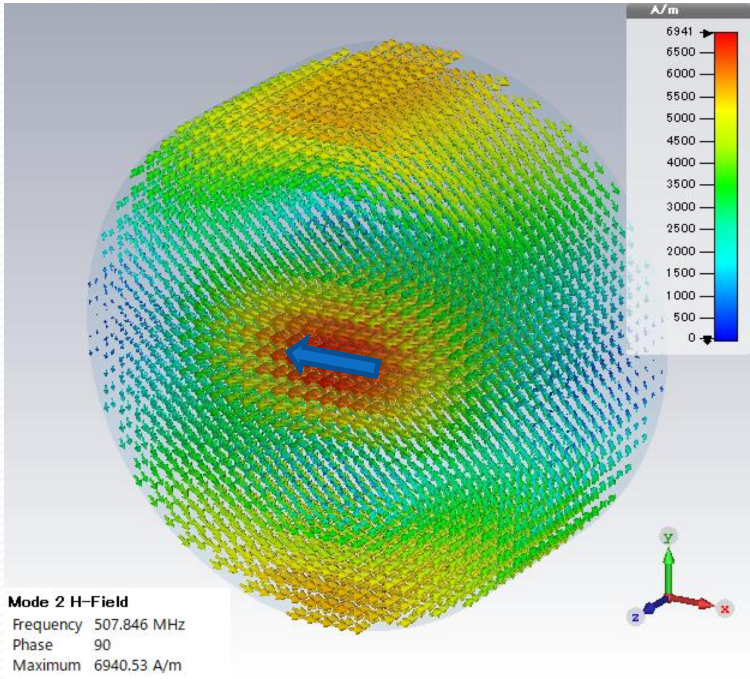




SOM

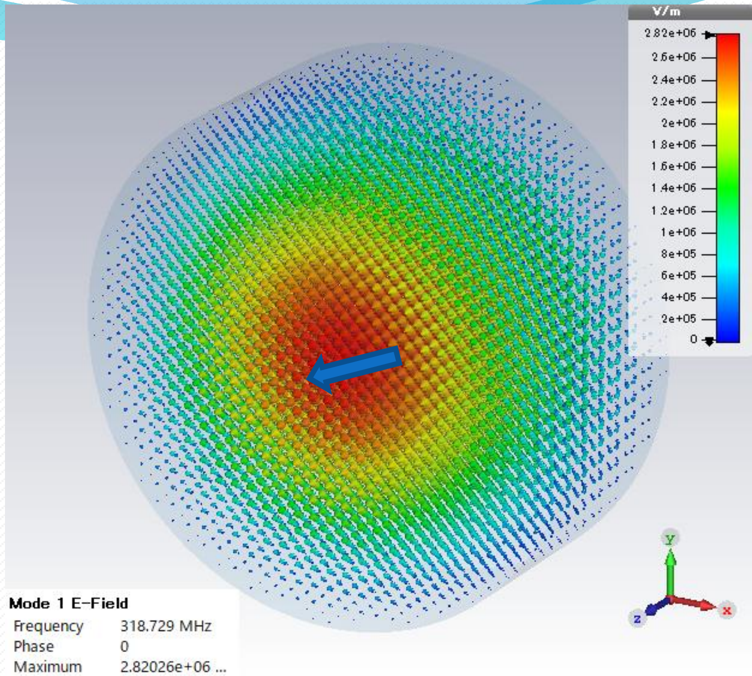
- Same order mode
  - Unwanted polarization
  - Vertical kick
  - Same frequency

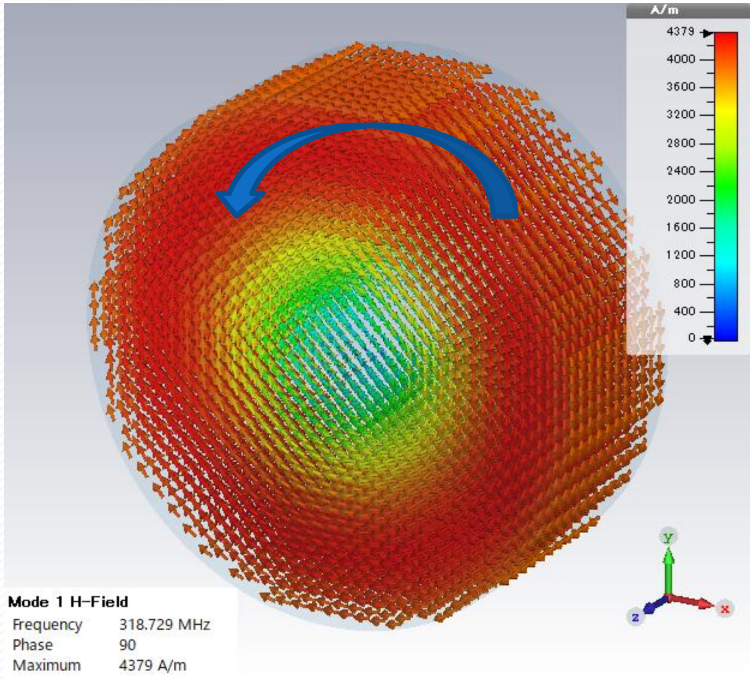




LOM

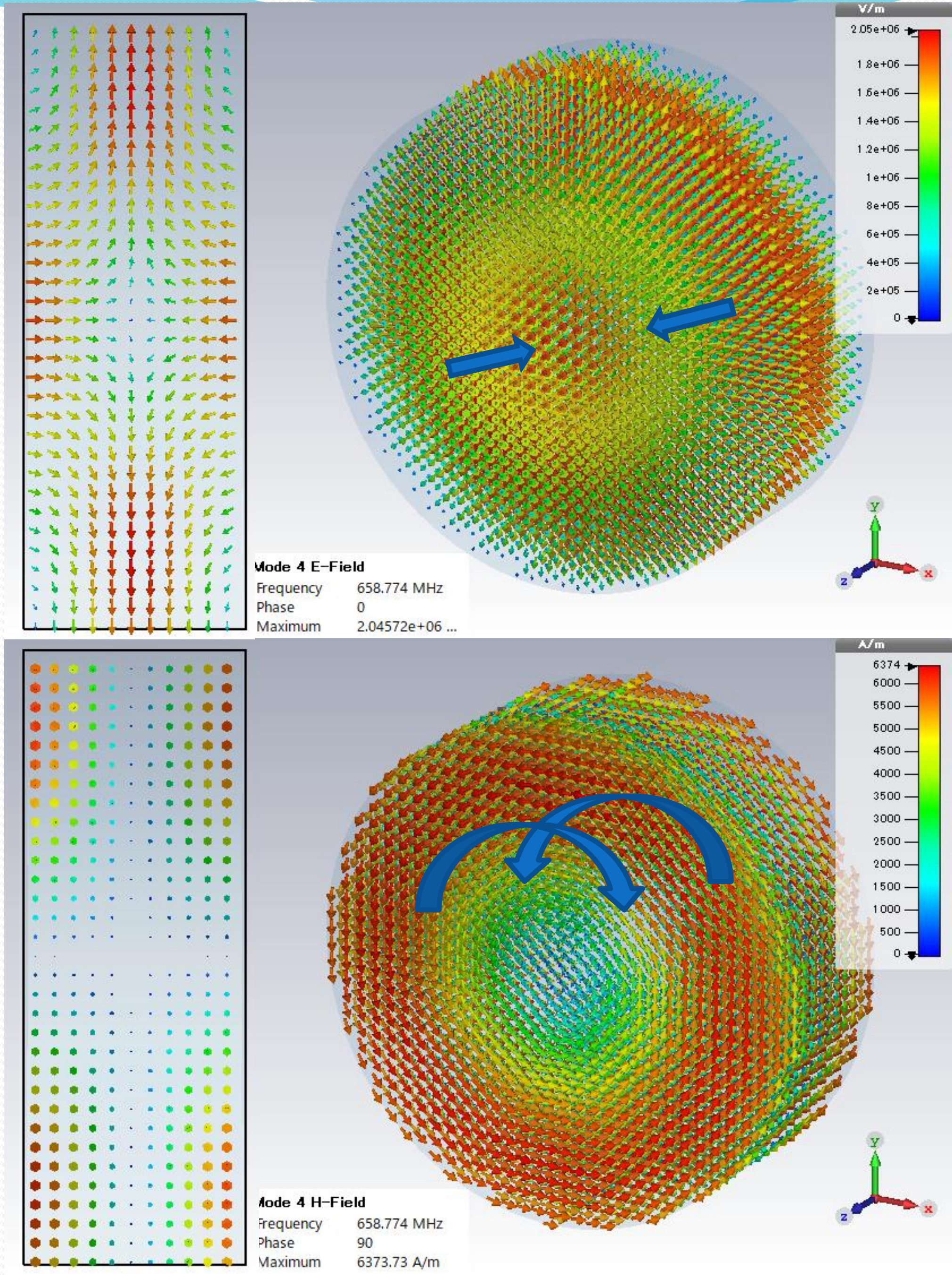
- Lowest order mode
  - Fundamental mode
- TM010 monopole mode
  - Lower frequency: 320 MHz
  - Accelerating mode
  - R/Q~100
  - Need to be heavily damped





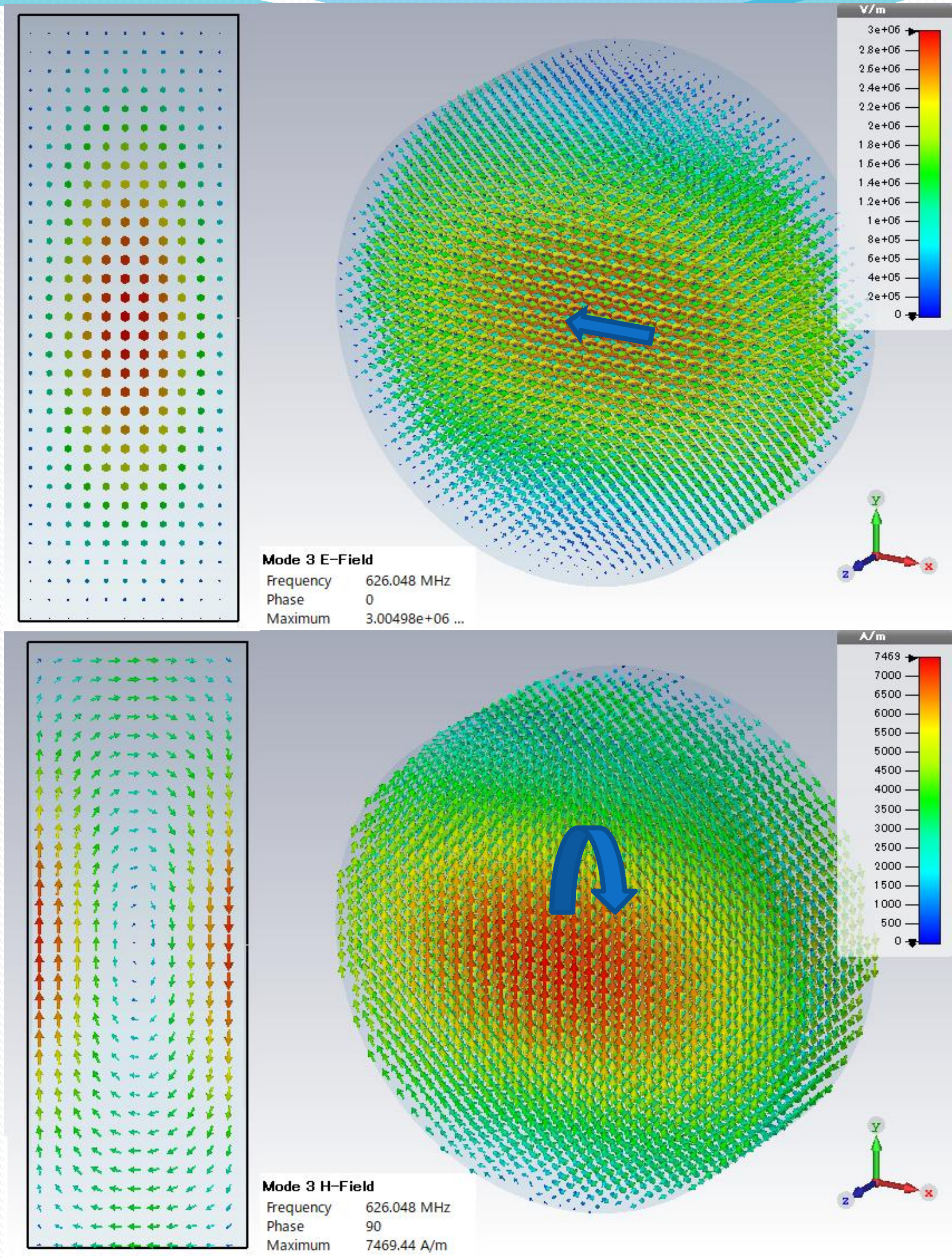
HOM

- Higher order mode
- TM011 monopole mode
- Accelerating mode
- Should be heavily dumped



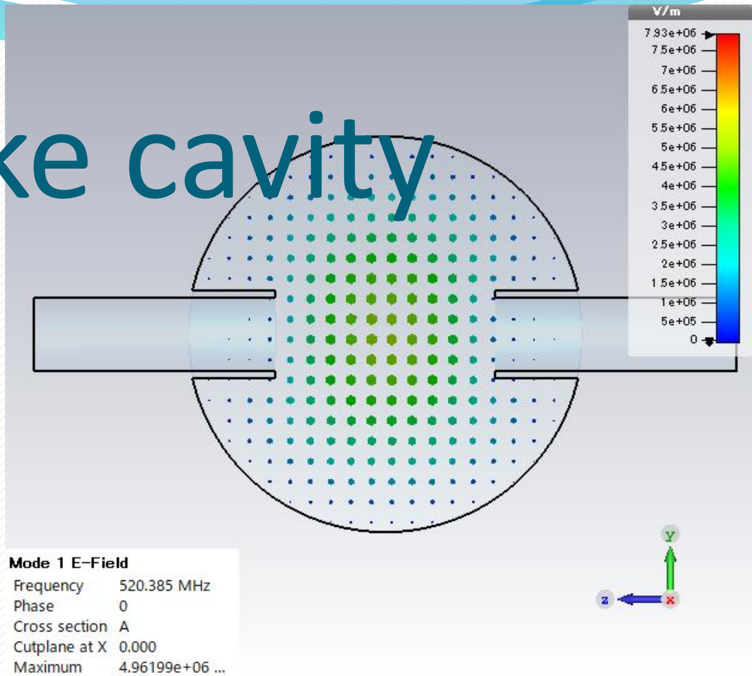
HOM, Crab cavity?

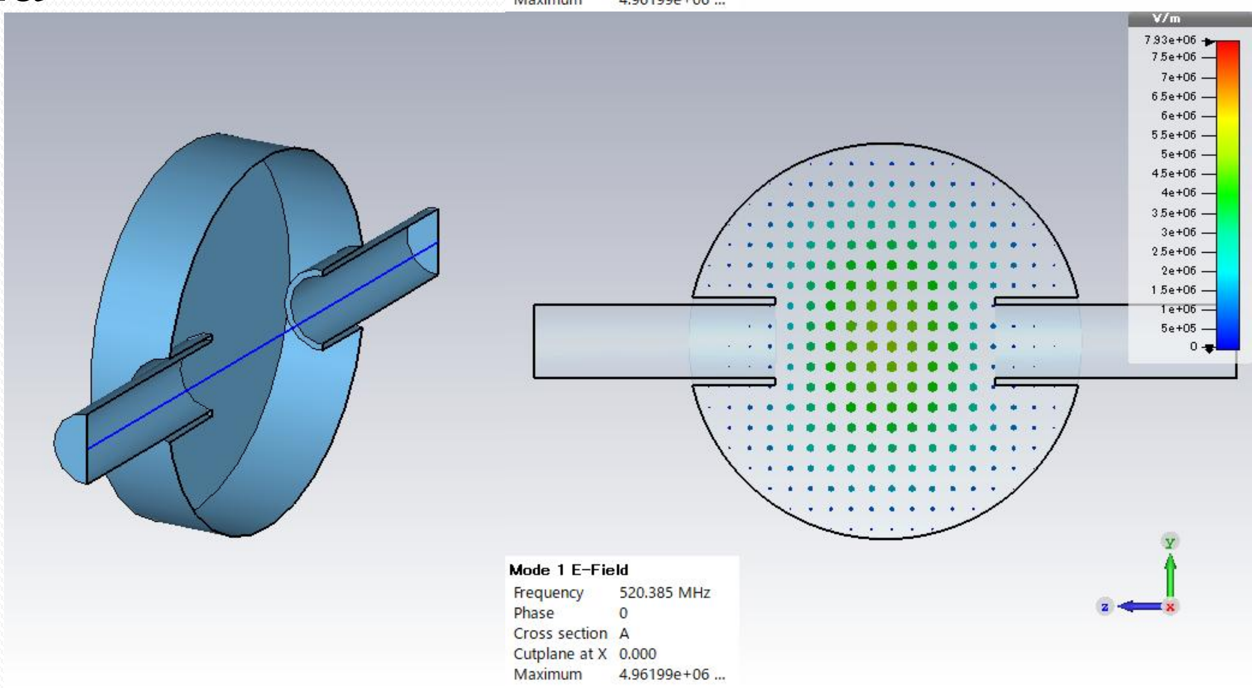
- TE111 dipole mode
- Crab cavity?
- Answer is No!
- Magnetic kick cancels electric kick
- Remember Panofsky-Wenzel theorem
  - No kick when Ez=0



We can design TE-like cavity

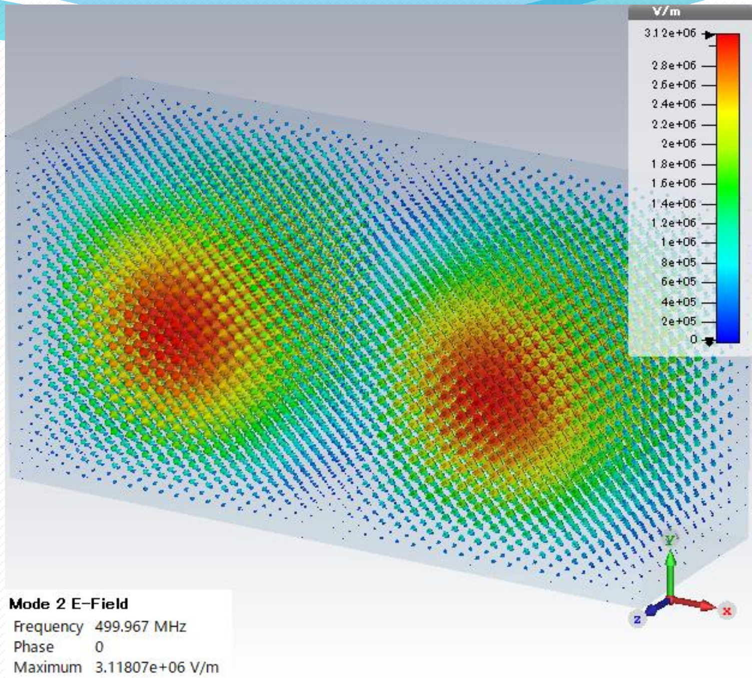
- TE cavity with beam pipe
  - Frequency: 500 MHz
- If the beam pipes shield the magnetic field
- We can obtain a horizontal kick
  - R/Q~40

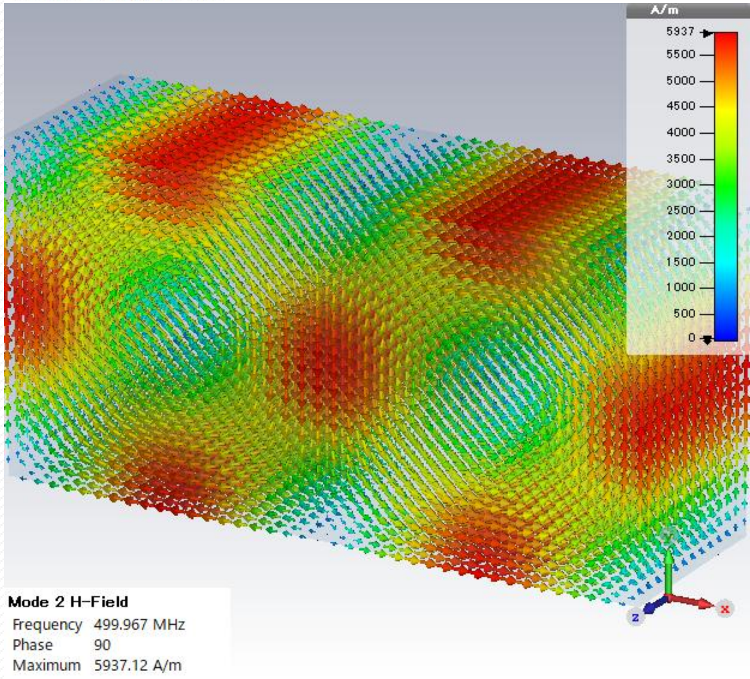


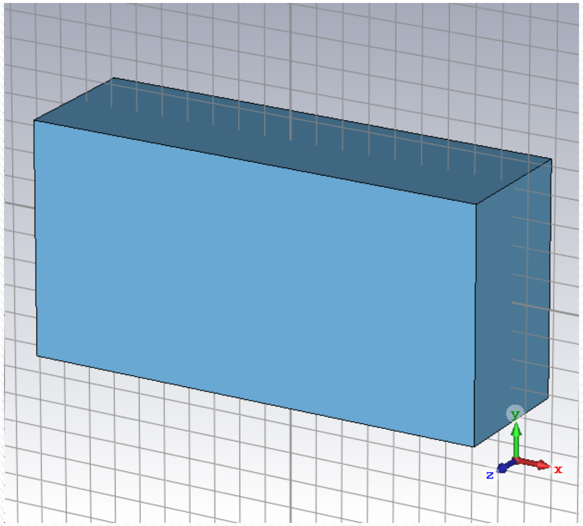


Rectangular cell

- Crabbing mode, TM210
- 500MHz

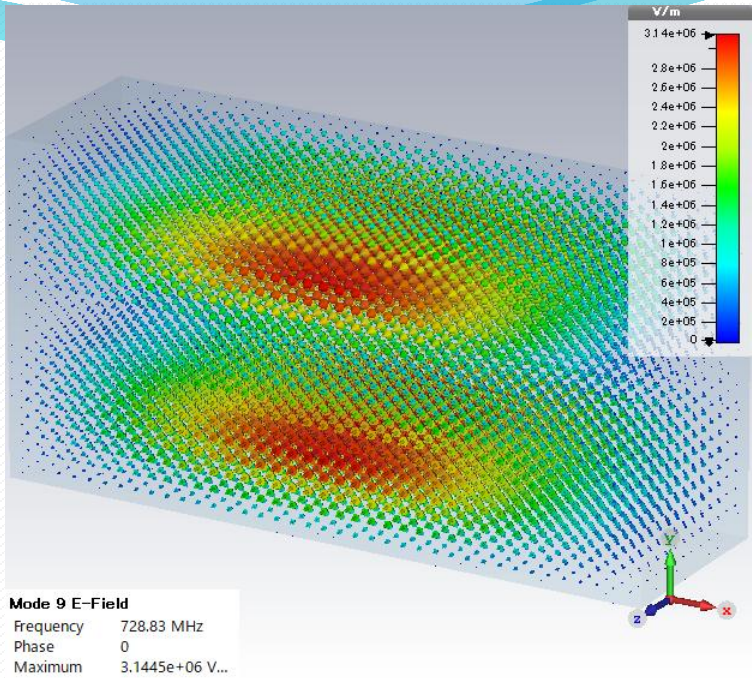


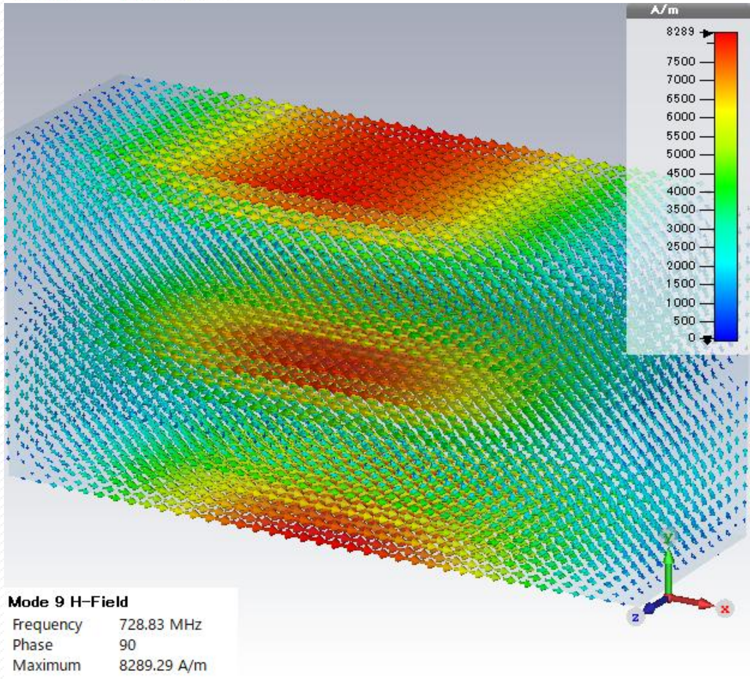




SOM

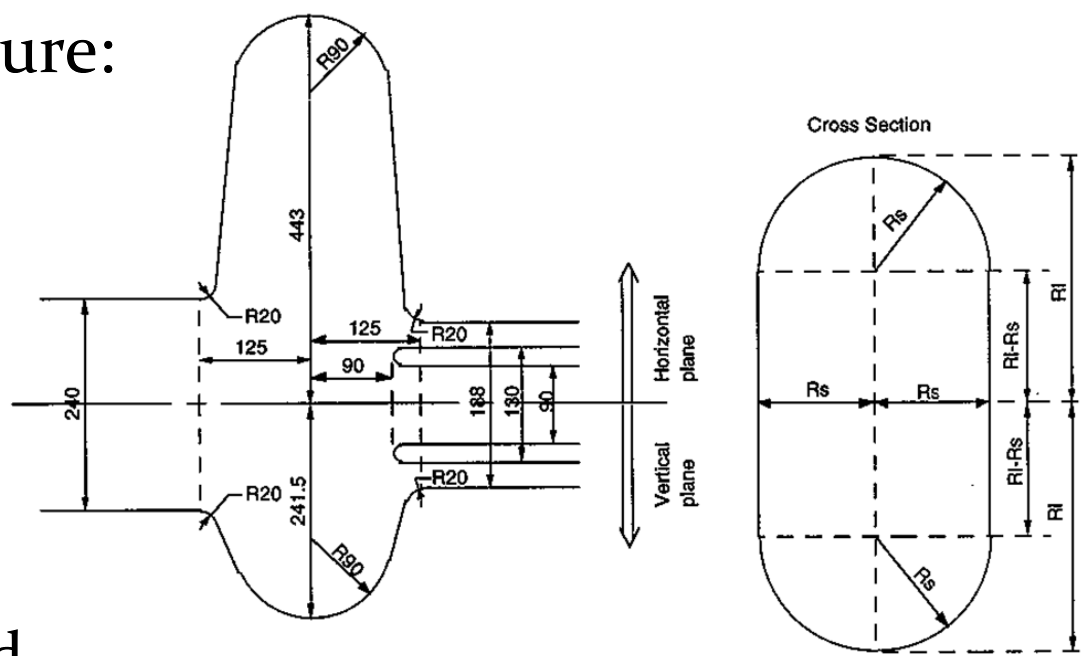
- TM120
- Vertical kicking mode
- Higher frequency than the crabbing mode
- We can separate vertical kicking mode from horizontal kick mode





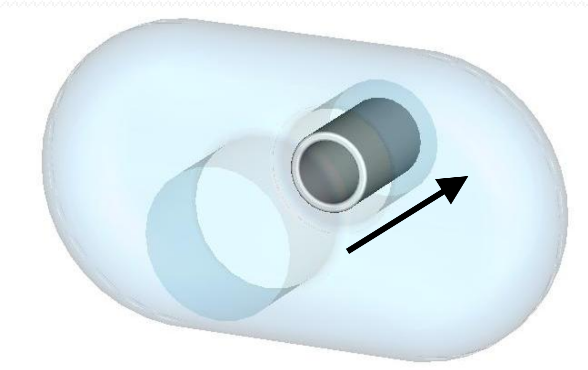
Crab cavity cell for KEKB

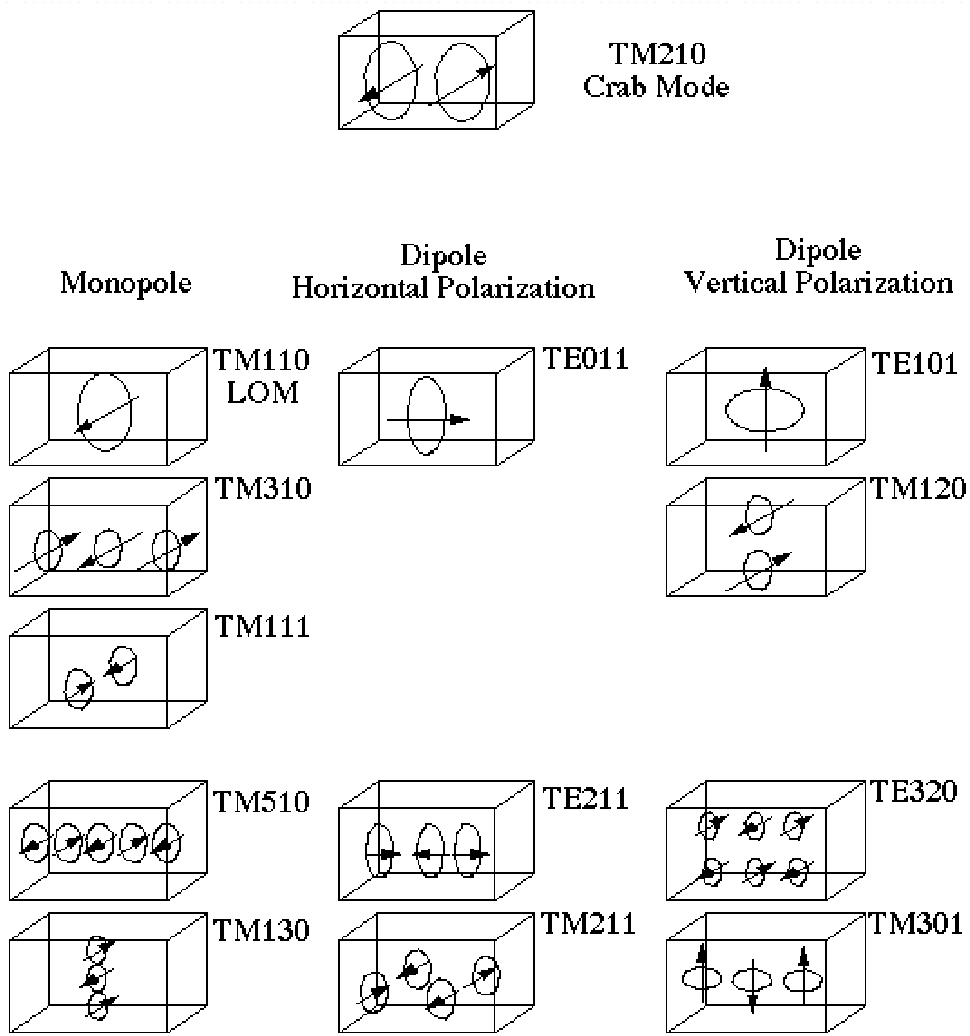
- Required voltage: 1.4MV
- Operating temperature: 4.4K
- Squashed cell
- Crabbing mode
  - TM210-like mode
  - 509 MHz
- Coaxial coupler
  - For LOM, SOM and HOMs damping



LOM, SOM and HOMs

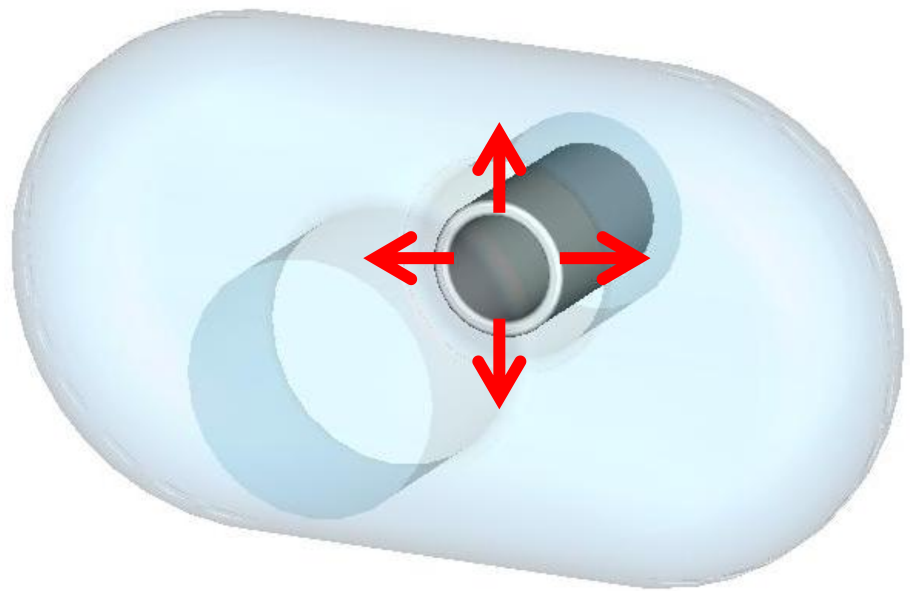
- Coaxial coupler propagates these modes
- TEM: No cut-off (monopole)
- TE11: fc=600 MHz (dipole)





LOM damping

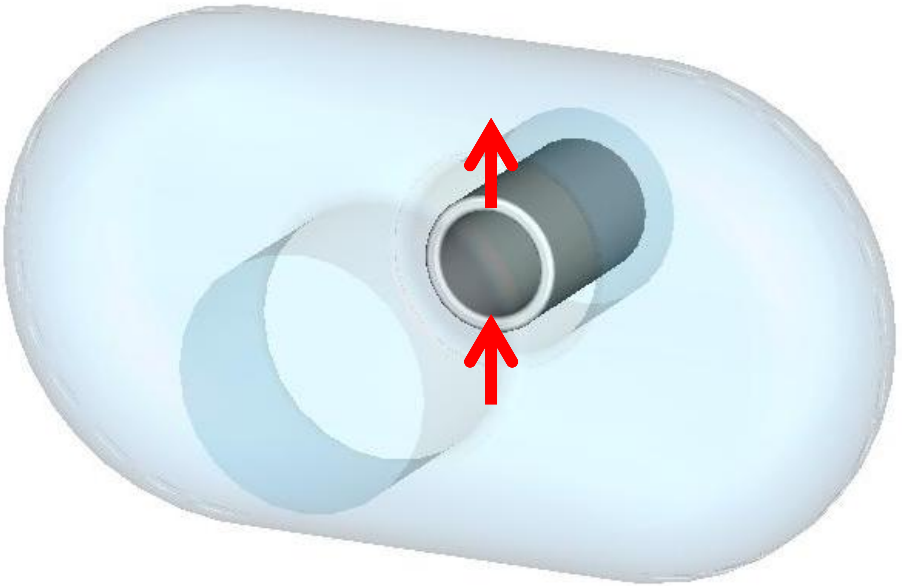
- Lowest order mode
  - Fundamental mode
  - Accelerating mode
  - R/Q~100
- LOM can propagate as the TEM mode in the coaxial coupler
- Coaxial ferrite RF absorber at the end of this coupler heavily damps this mode
  - Q~100



TEM mode in the coaxial coupler

SOM damping

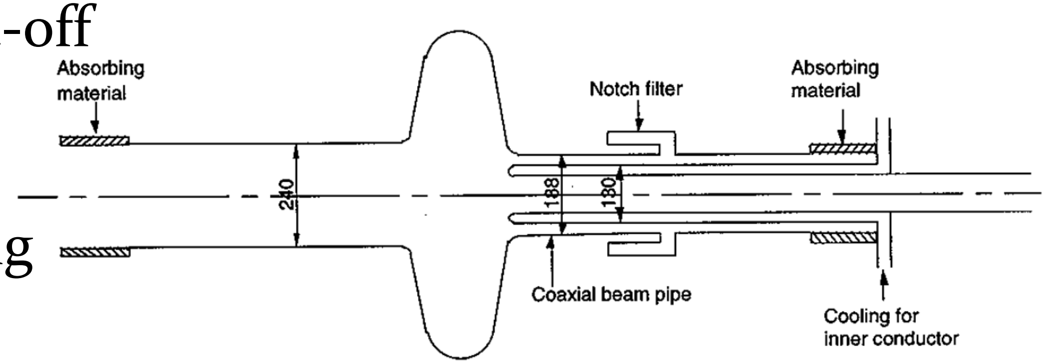
- Same order mode
  - Another polarization of crabbing mode
  - Frequency above 600 MHz by squeezing cavity cell
- SOM can propagate as the dipole mode (TE11) in the coaxial coupler
  - The cut-off frequency is 600 MHz
  - Above the crabbing mode
  - Blow the SOM mode



TE mode in the coaxial coupler

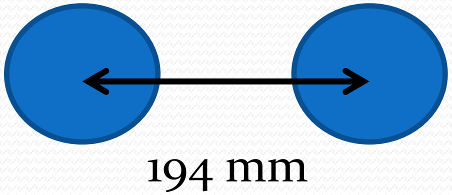
HOM damping

- Coaxial coupler and coaxial ferrite absorber
  - Monopole modes
  - Dipole mode above cut-off
  - Heavily damp those modes
  - Notch filter for crabbing mode rejection
- Large beam pipe and ferrite RF absorbers
  - Monopole>900MHz
  - Dipole>750MHz



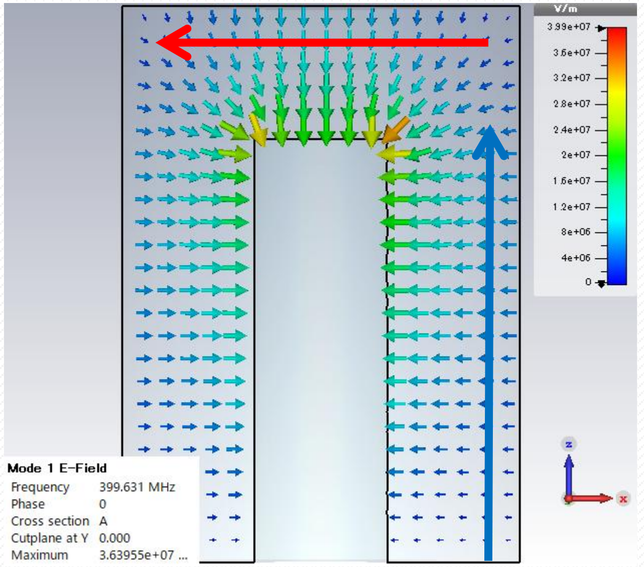
Compact crab cavities for LHC

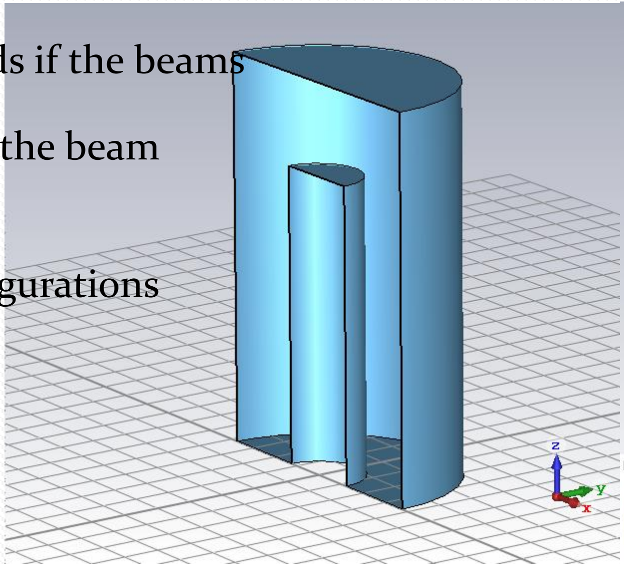
- Required for LHC application
- Required voltage: 3MV
- Frequency: 400 MHz
- Operating temperature: 2K
- Narrow space between beam pipes
  - Nominal beam separation: 194 mm
  - Beam pipe diameter: 84 mm
- Can not use pill box type cavities
  - 800MHz cavity clears size problem but the high frequency gives non-linear kick

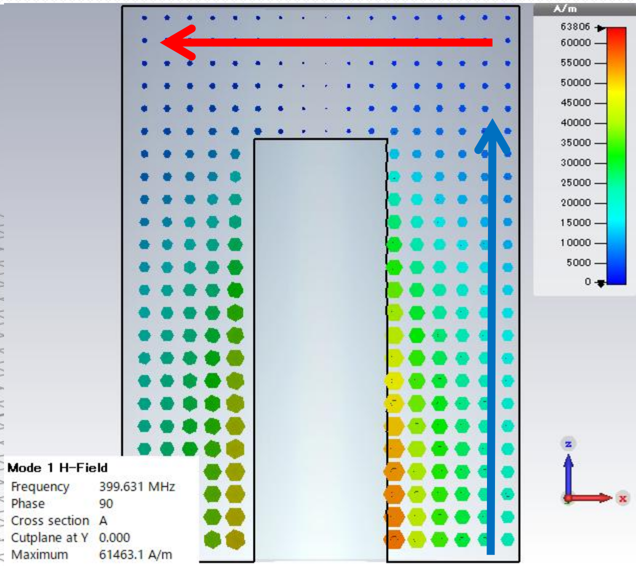


Quarter wave resonator

- Compact cavity
- Used for low beta beam accelerations
  - Low frequency application
  - If you make a 200MHz pill box cavity, its diameter should be 1 m.
- Maximum E field exists at the open end of an inner conductor
  - Deflection by the E-field if the beams pass along the red line
- Maximum H field exists at the short end of an inner conductor
  - Deflection by the H fields if the beams pass along the blue line
  - E fields also contributes the beam deflection
- Beam acceleration
  - 2 or 4 symmetrical configurations avoid acceleration

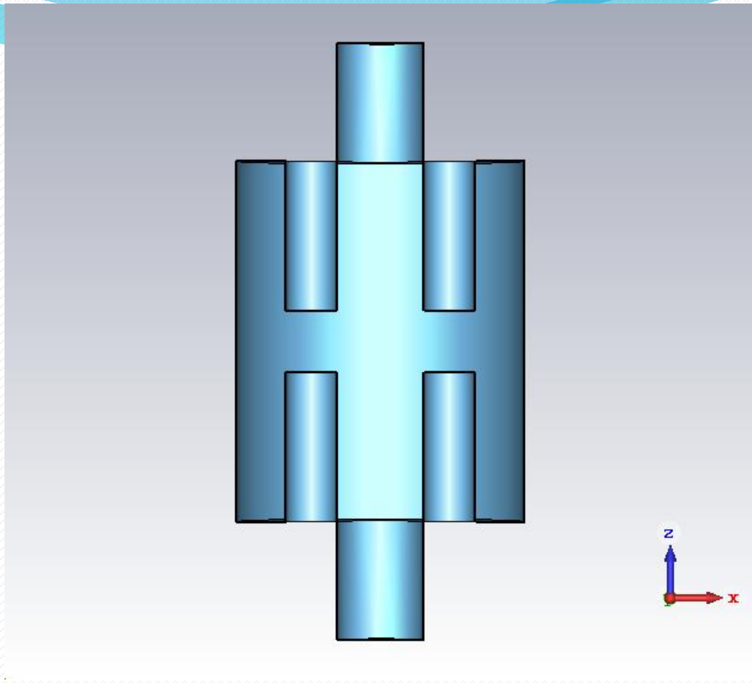


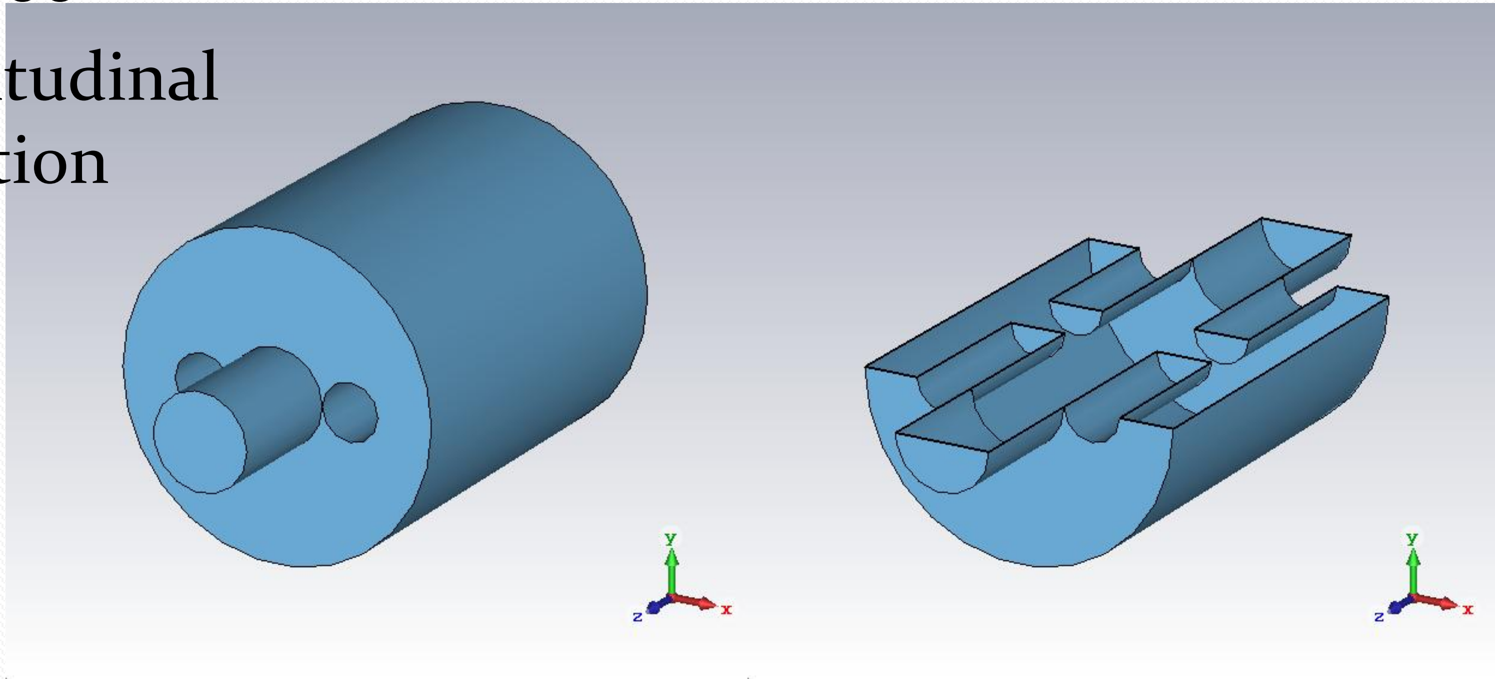




Four QWR configuration 4RCC

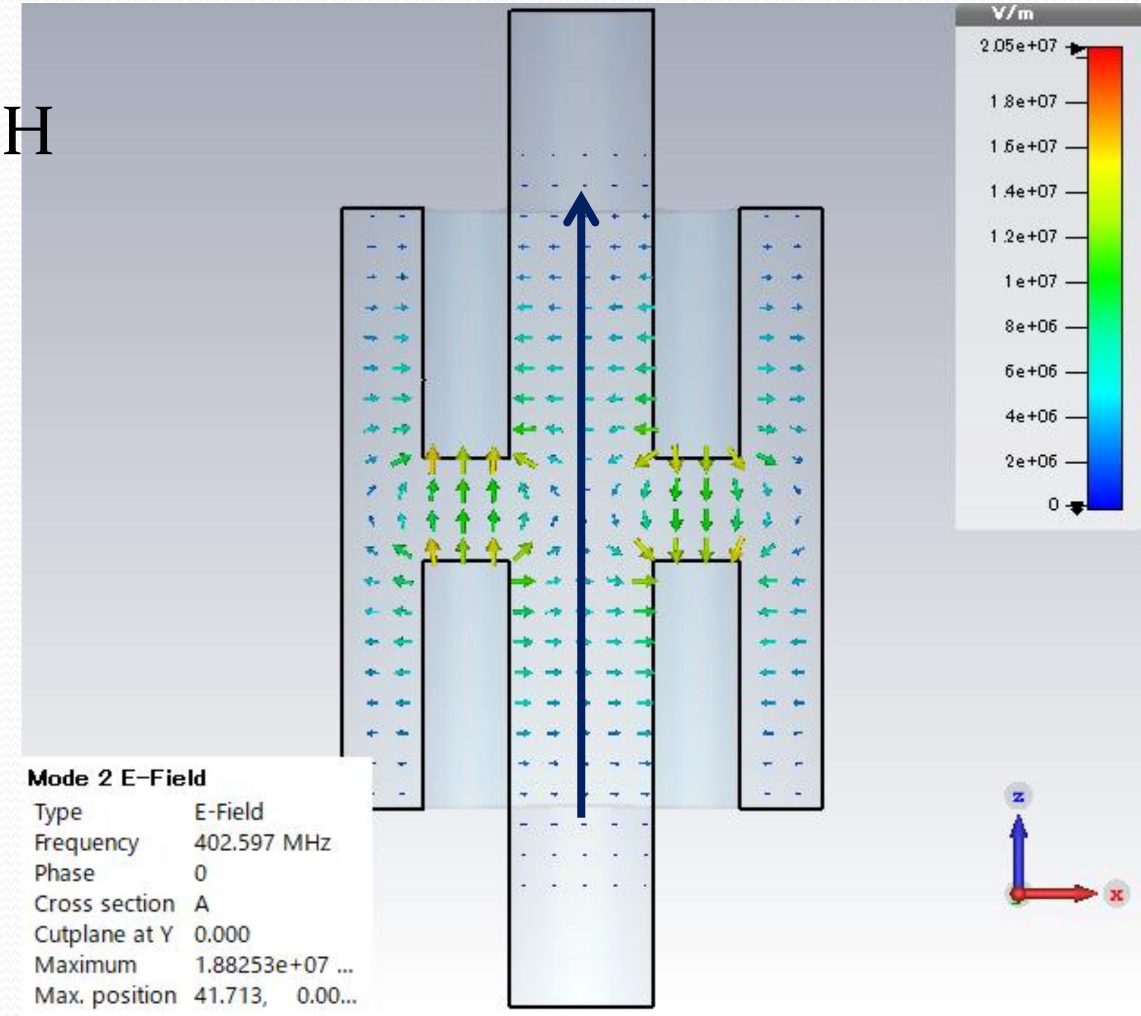
- Consists of four QWRs
- Use the E and B fields for beam deflection
- R/Q～1000
- No longitudinal acceleration
- Not FM





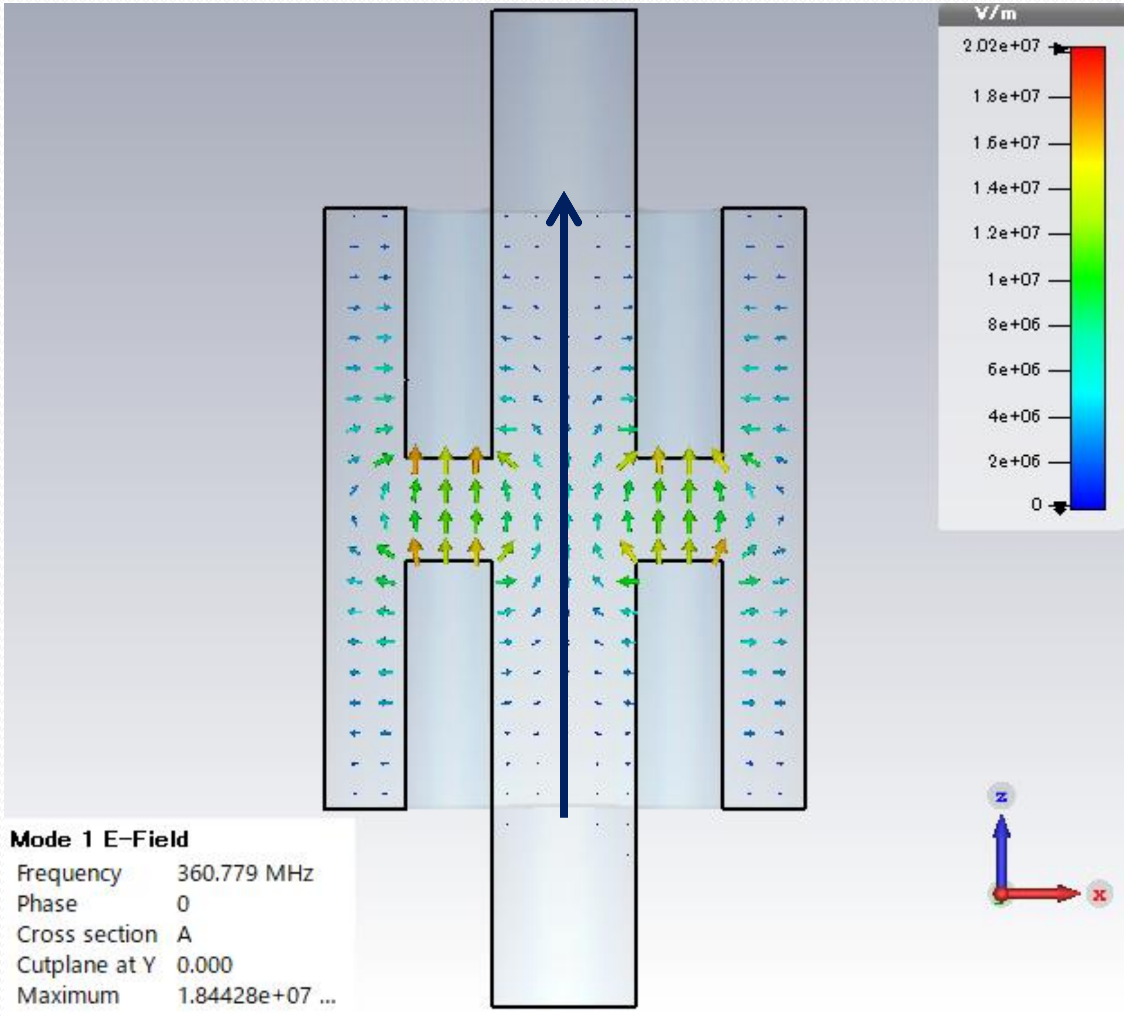
Crabbing mode

- 400 MHz
- Deflecting by E and H fields
- No longitudinal acceleration
- R/Q~1000



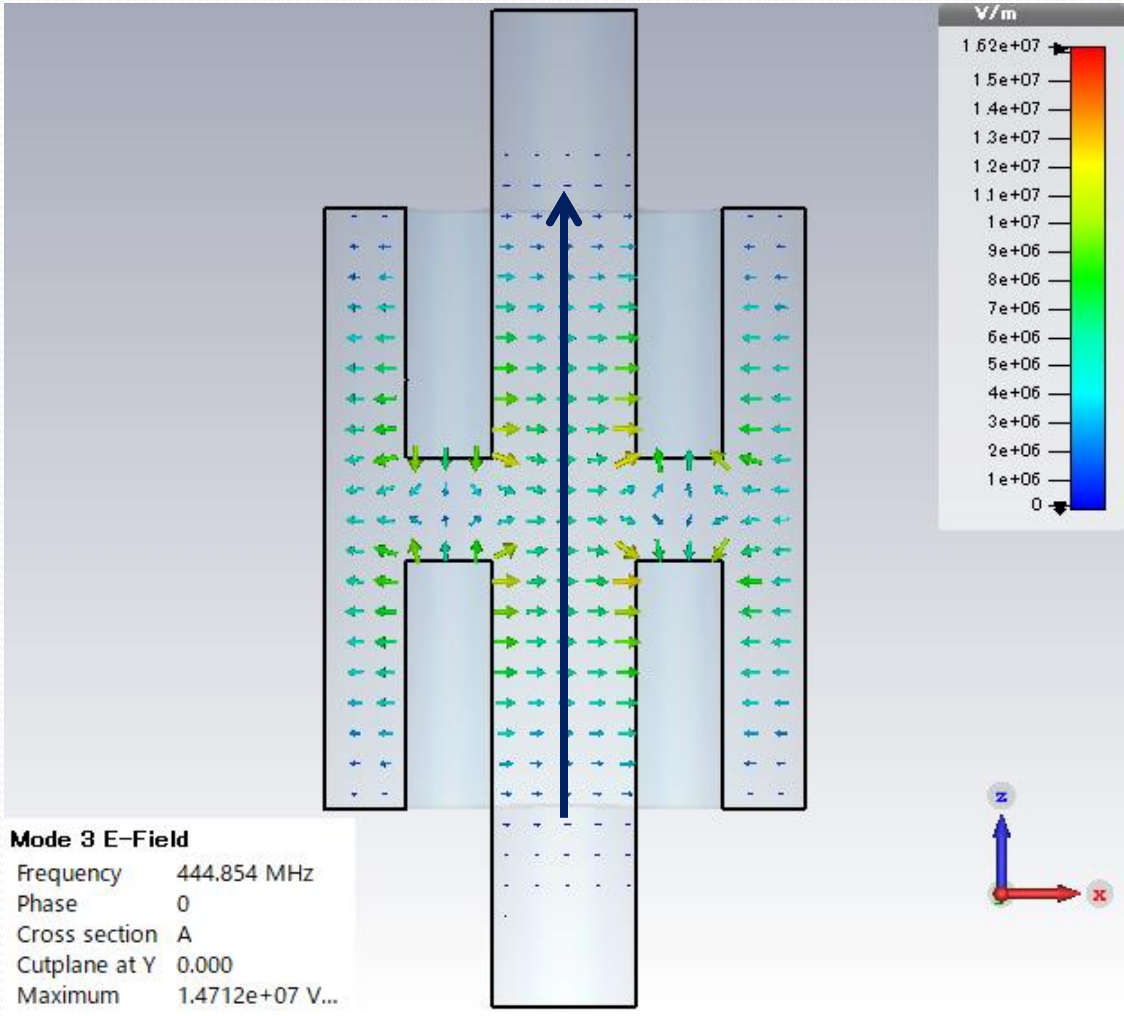
Fundamental mode

- 360 MHz
- Accelerating mode



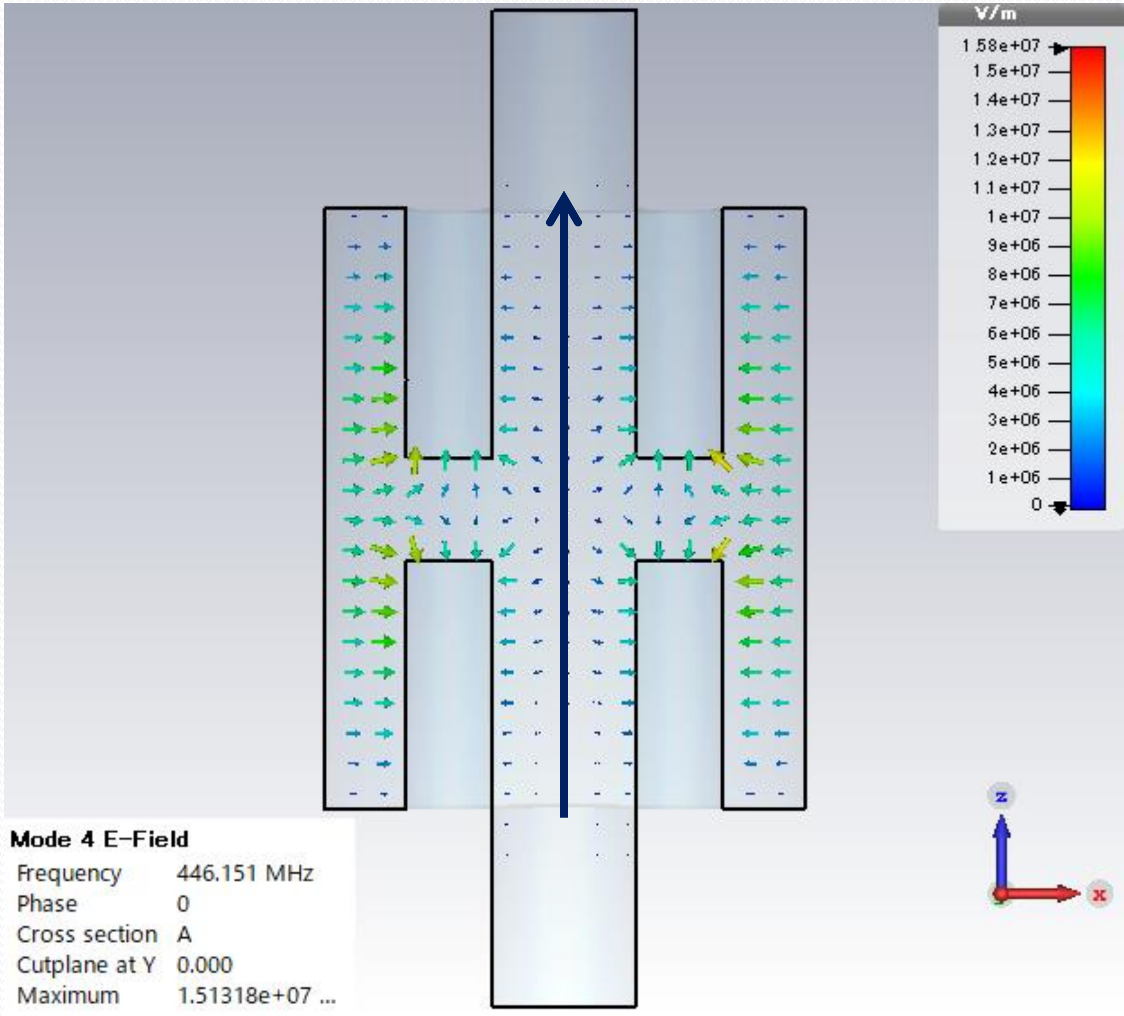
1st HOM

- 445 MHz
- Deflecting mode

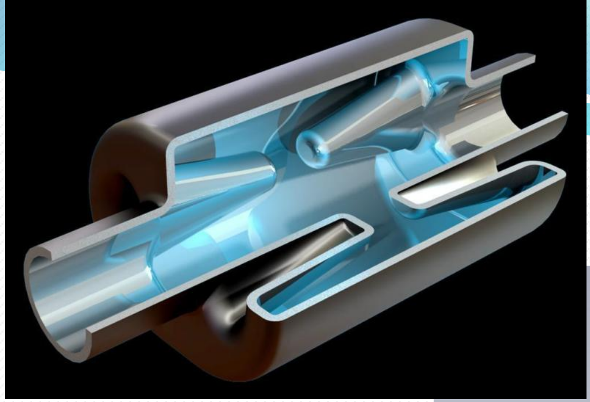


2nd HOM

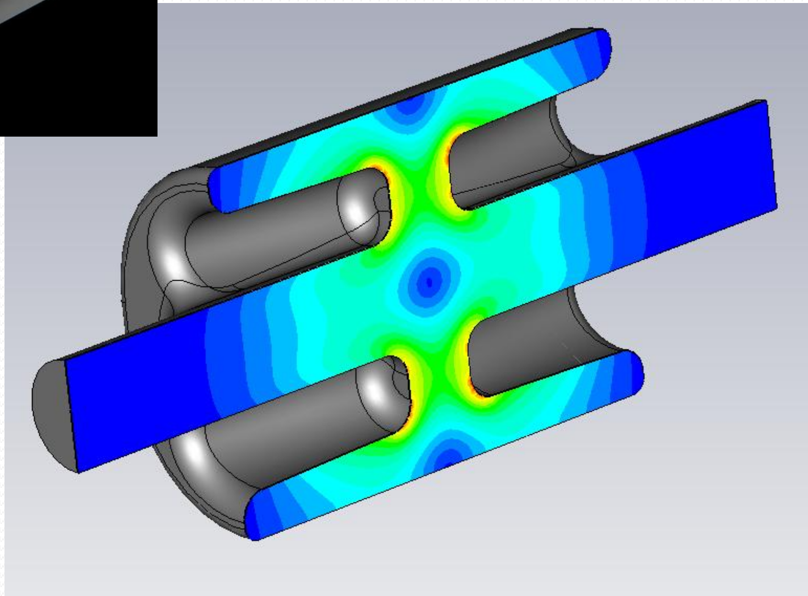
- 446 MHz
- Accelerating mode

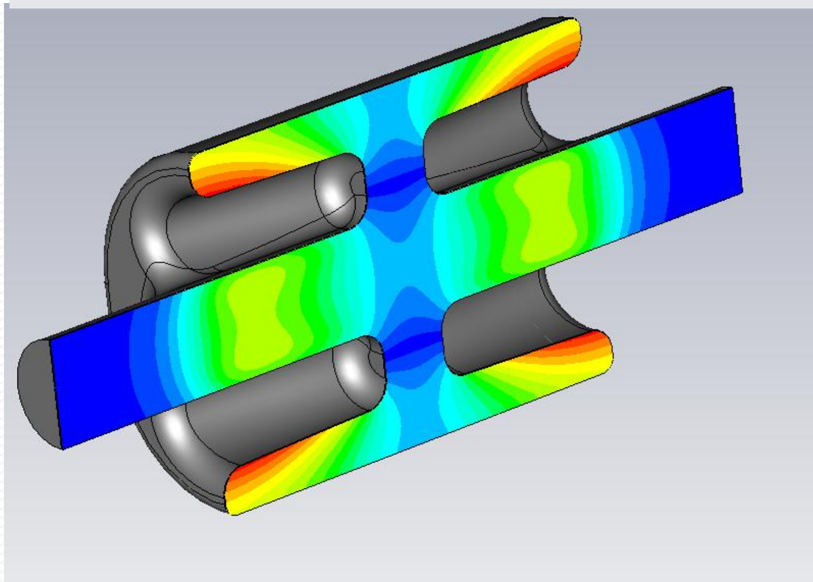


4RCC(UK)



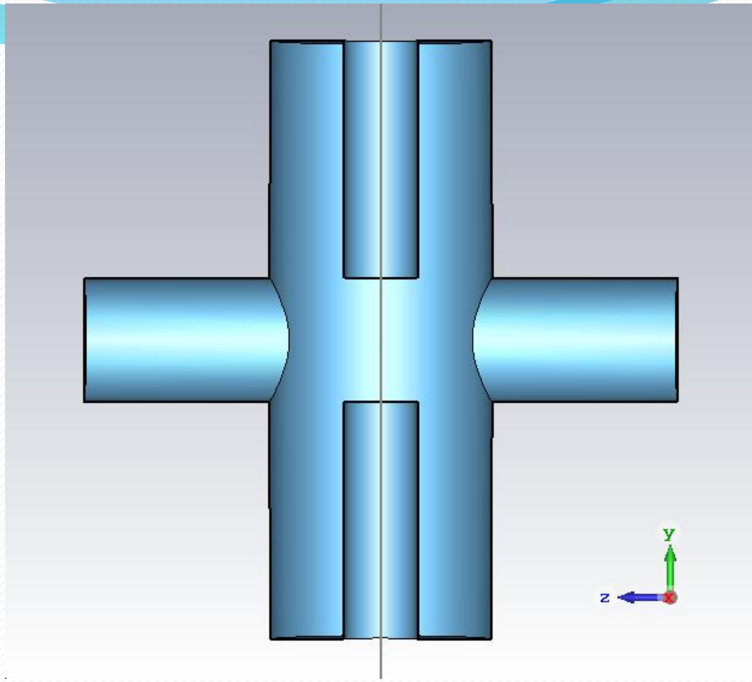
- 4-rod crab cavity
- ULANC
- Cavity size: 143x118 mm^2
- Cavity length: 500 mm
- Peak E field: 32MV/m
- Peak B-Field 61 mT
- R/Q: 915
- FM: 370 MHz
- Not fundamental mode

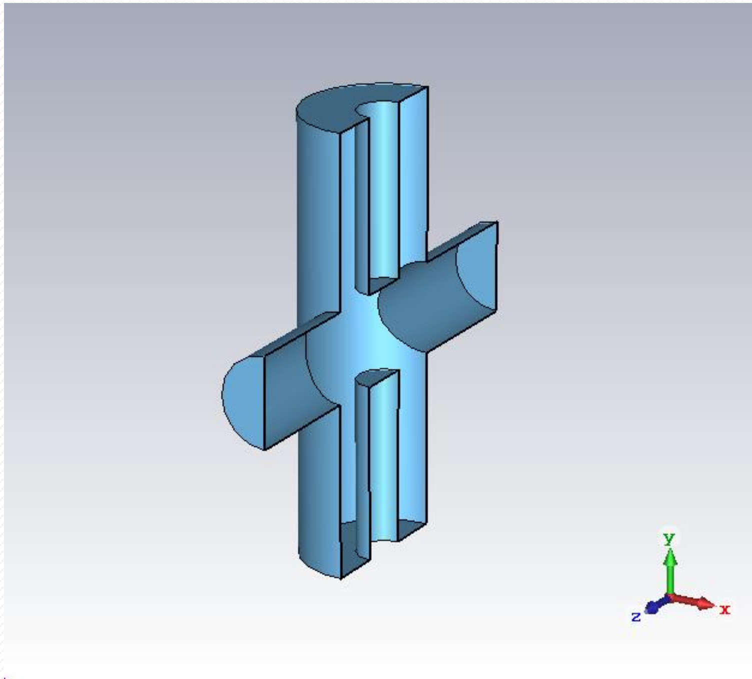




Two QWR configuration DQW

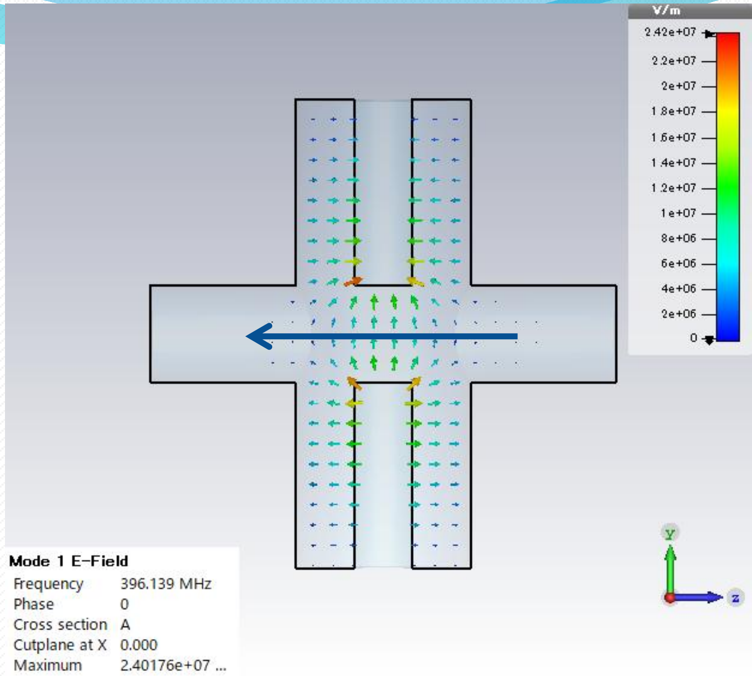
- Consists two QWRs
- Mainly use the E-field for beam kick
- No longitudinal acceleration
- FM

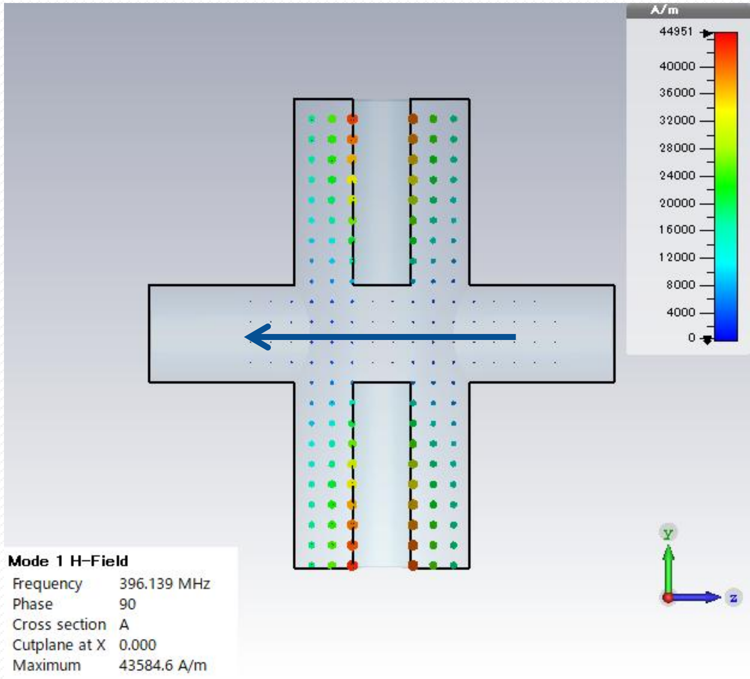




Crabbing mode

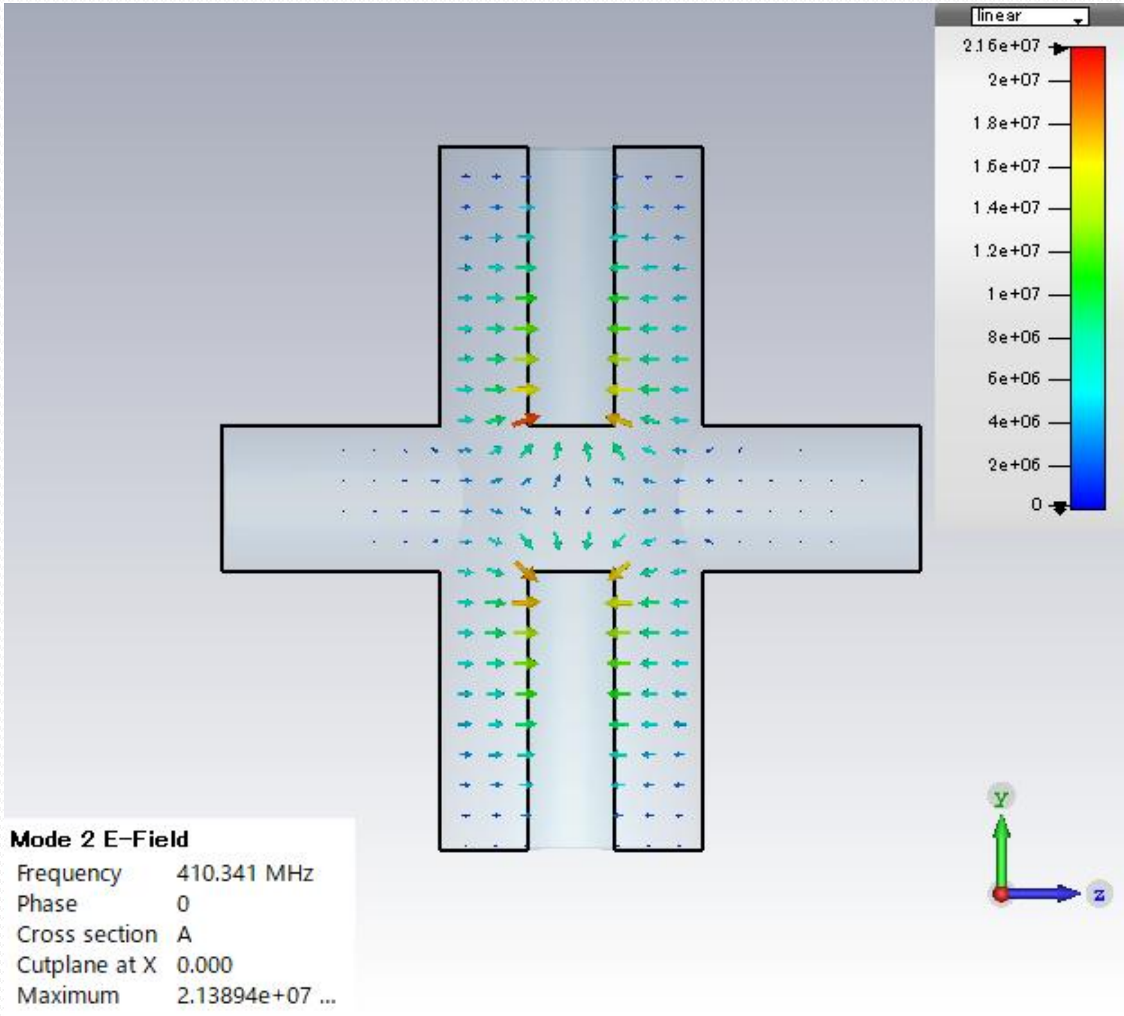
- 400 MHz
- Deflecting mainly by the E field
- No longitudinal acceleration





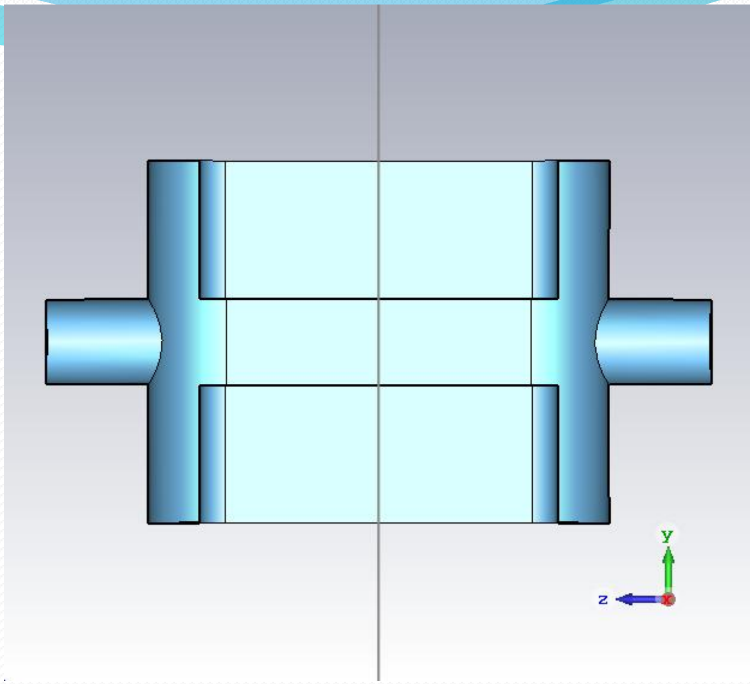
HOM

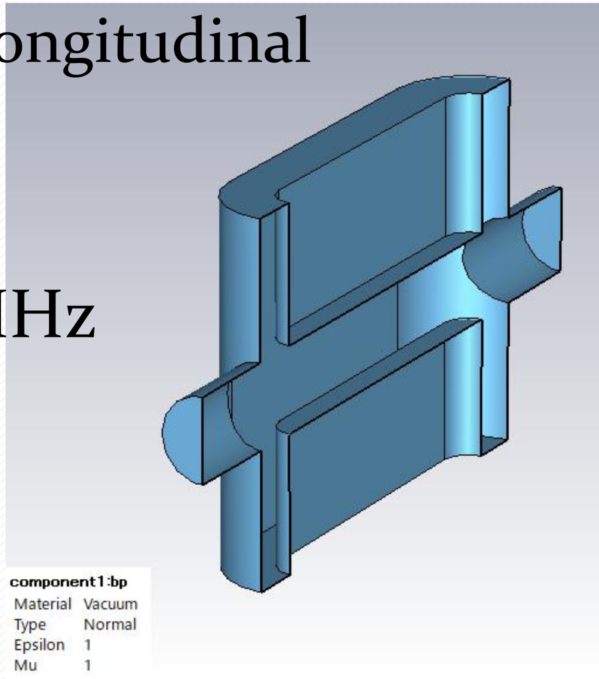
- 410 MHz
  - Near the crabbing frequency
  - Well separate after longitudinal length optimization
- Accelerating mode

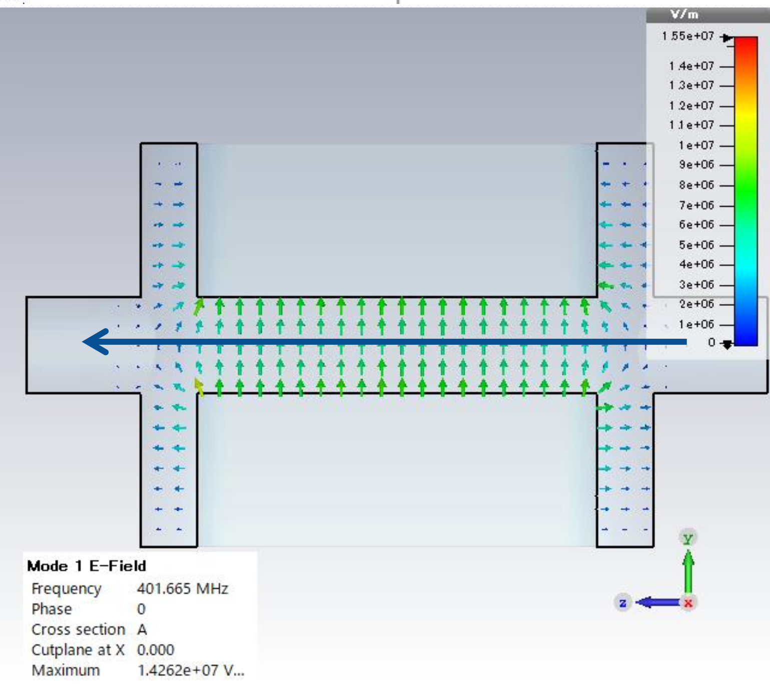


Longitudinal length

- To obtain larger kick voltage
  - Deflecting area has to be increased
  - Optimize longitudinal length~$\lambda/2$
- R/Q～500
- HOM: 457 MHz

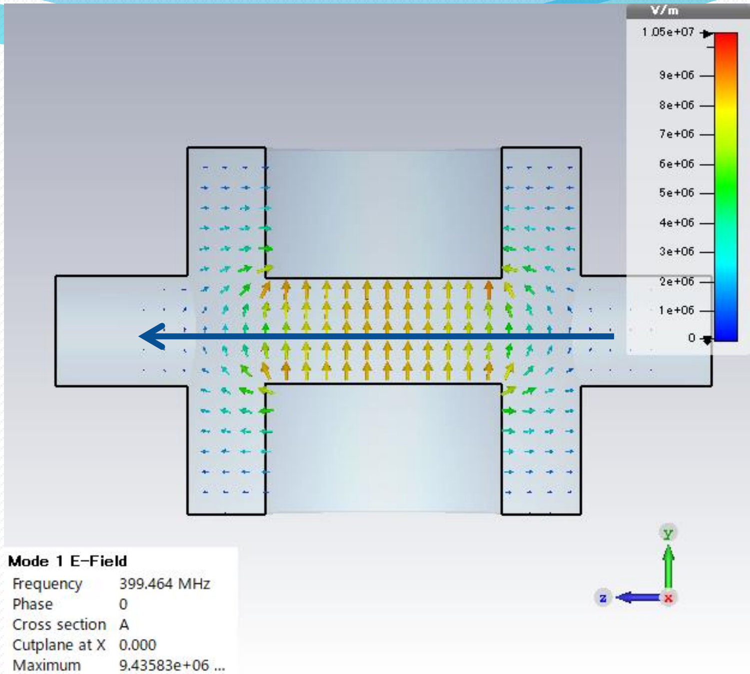


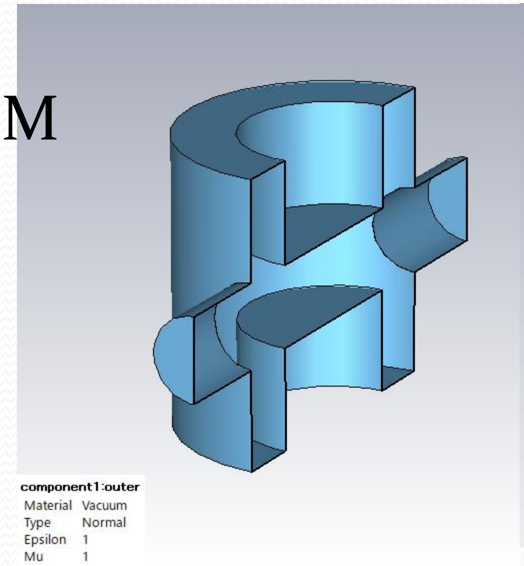


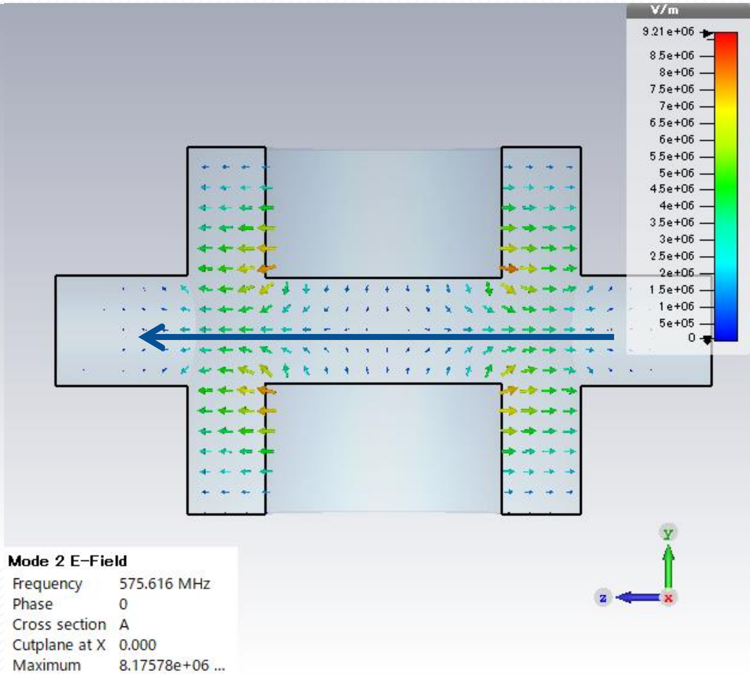


Transverse length

- Increase of transverse length
  - Results in shorter inner conductor
    - 160 to 100 mm
  - Increasing HOM frequency
    - 575 MHz
    - Well above FM

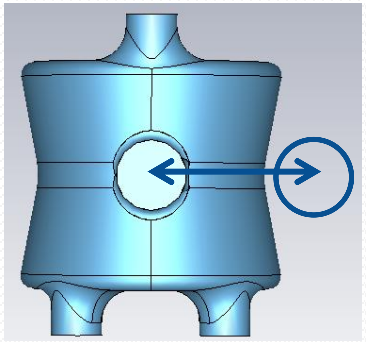


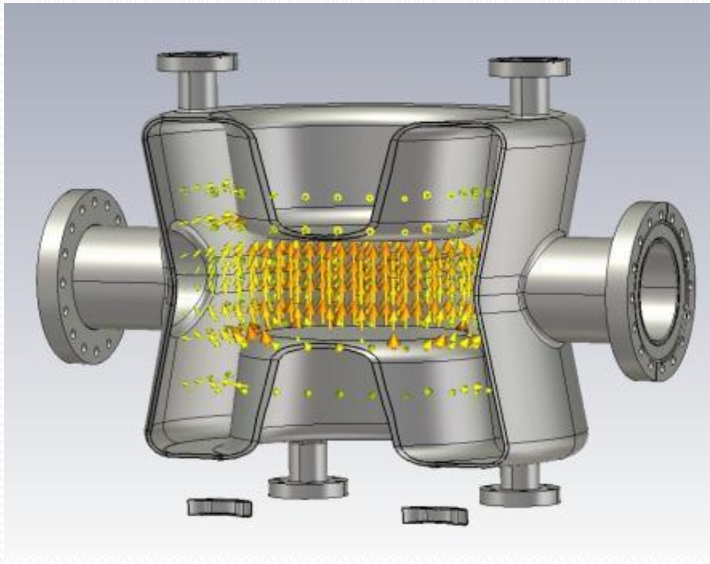


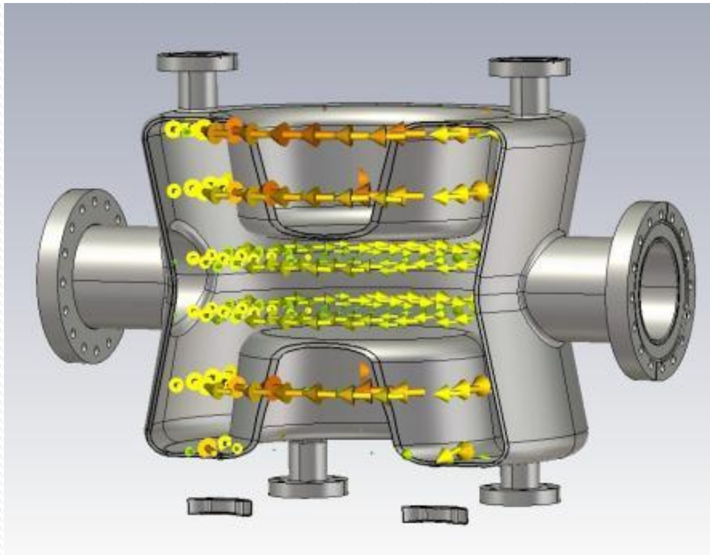


DQW

- Double quarter wave cavity
- BNL
- Vertical kick
- FM
- Cavity size: 142x122 mm²
  - Need slim waist to provide space for the other BP
- Cavity length: 384 mm
- Peak E field: 43MV/m
- Peak B-Field 61 mT
- R/Q: 345
- HOM: 575 MHz

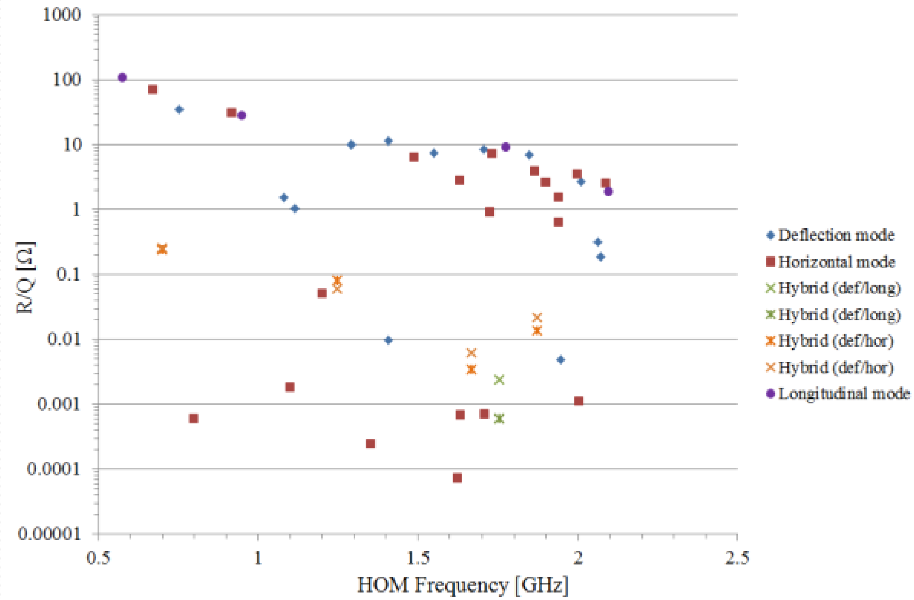


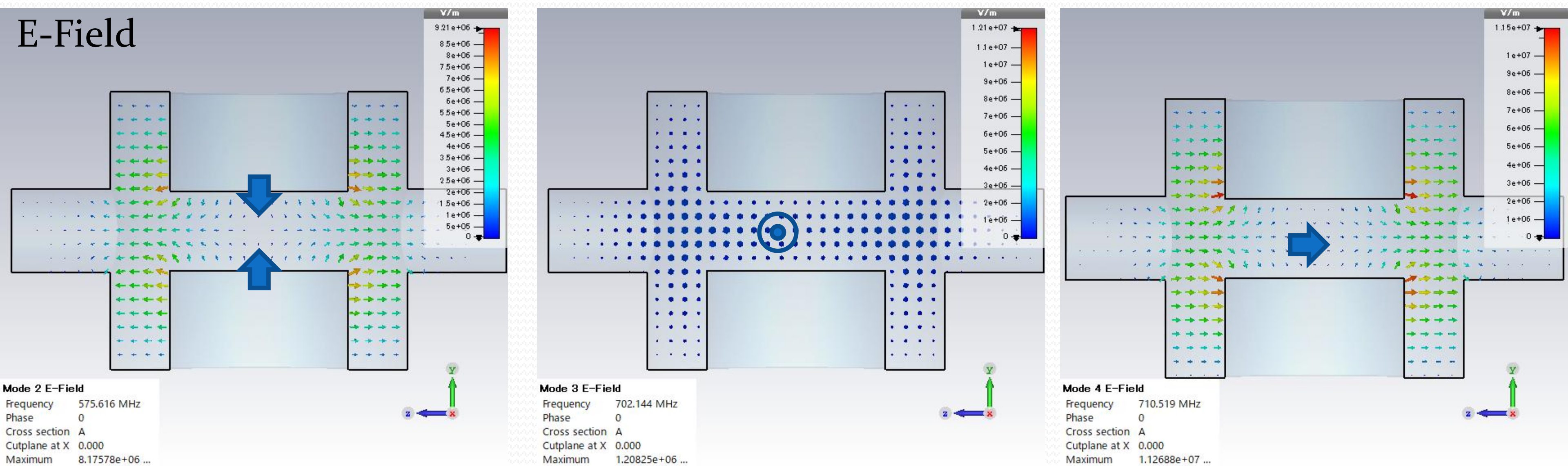




HOMs

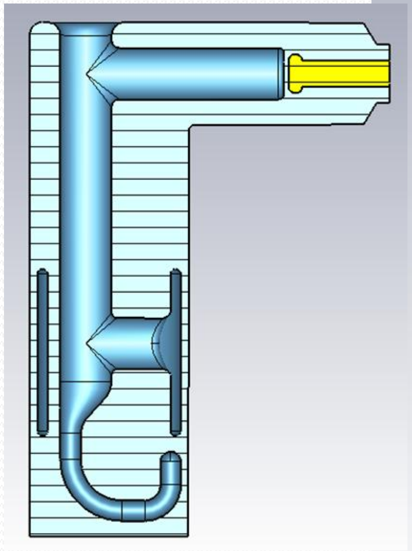
- HOMs are well separate from FM
  - Above 550 MHz

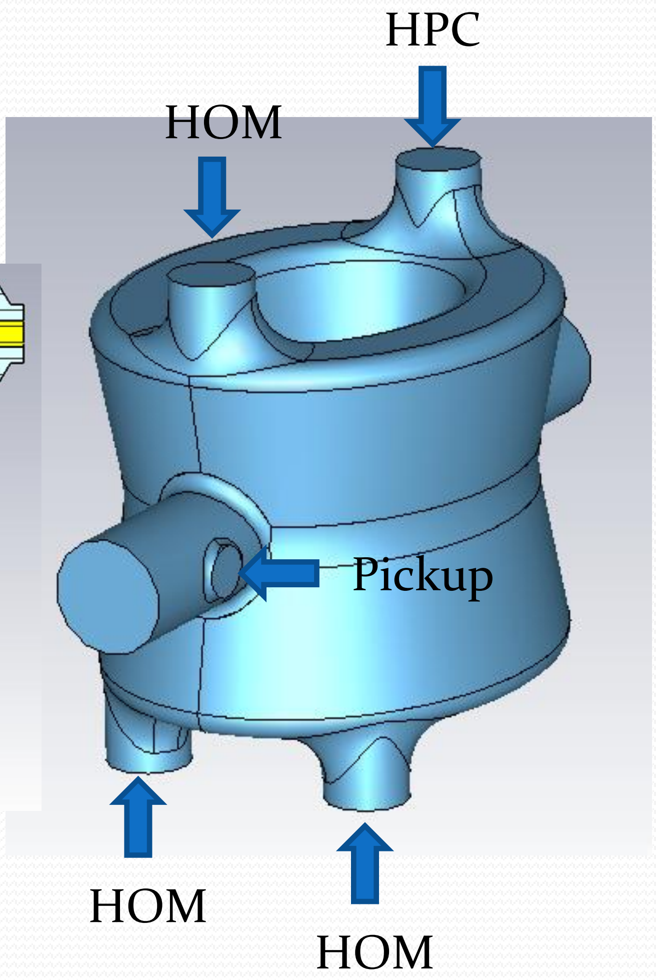




HOM damping

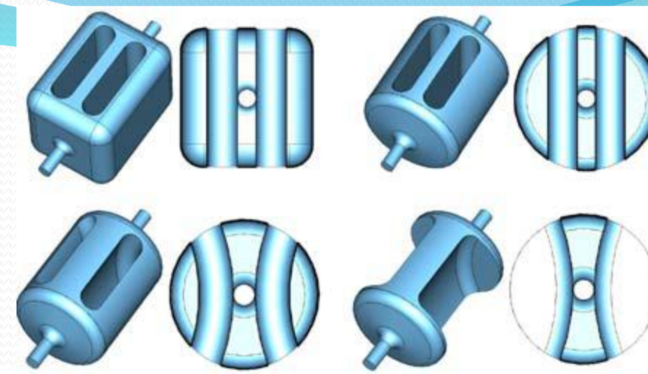
- HOMs are extracted by couplers
  - Hook-type coupler
  - Band stop filter at 400MHz
  - L-type high pass filter above 570MHz

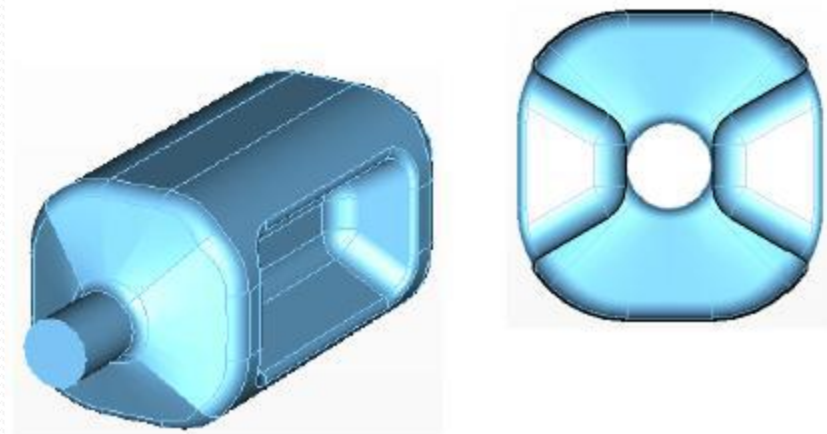


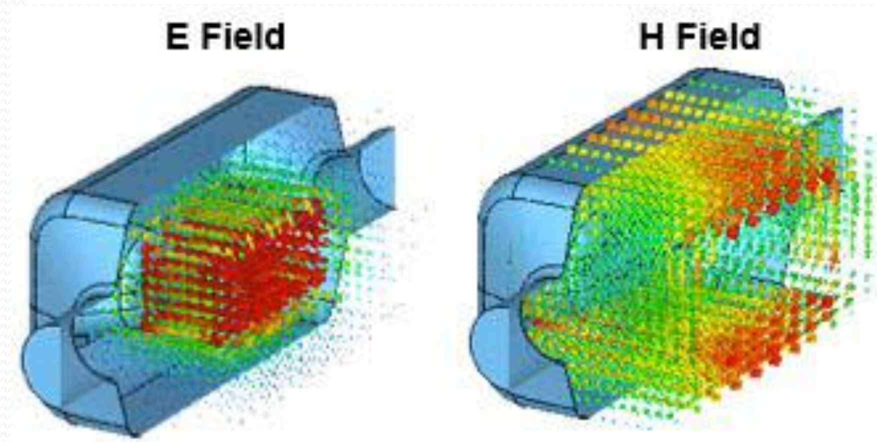


RFD

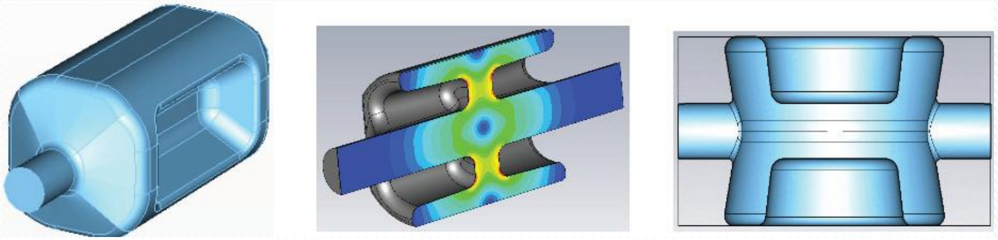
- RF deflector
  - Same topology
  - Evolved from the parallel bar geometry
- ODU/JLAB/SLAC
- Horizontal kick
- FM
- Cavity size: 147x147 mm^2
- Cavity length: 600 mm
- Peak E field: 33MV/m
- Peak B-Field 56 mT
- R/Q: 287
- HOM: 580 MHz







Compact CC

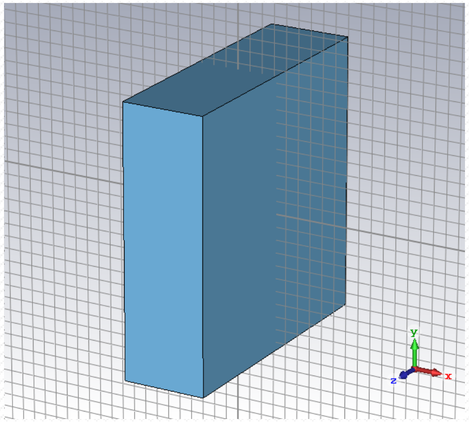


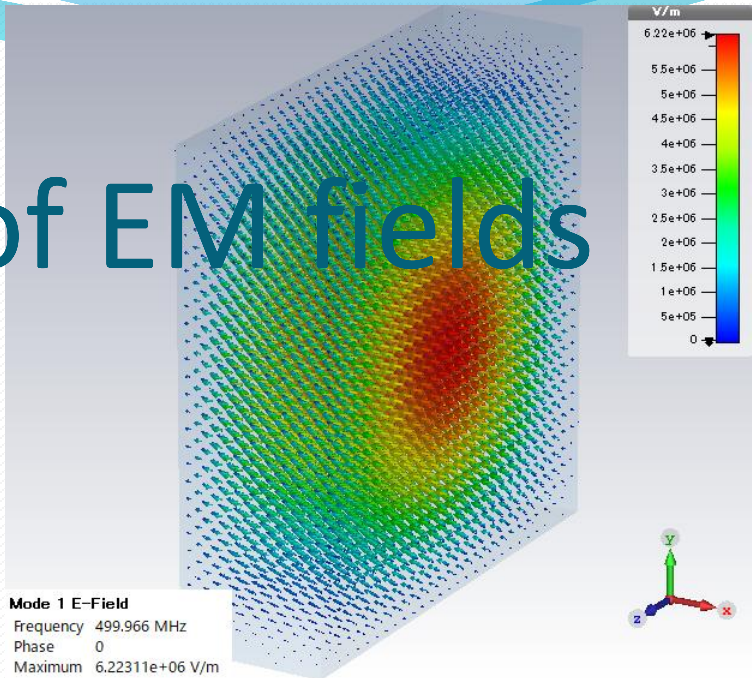
400MHz, 3MV	RFD	4RCC	DQW
Cavity radius (mm)	147.5	143/118	142/122
Cavity length (mm)	597	500	380
Beam pipe radius (mm)	42	42	42
Peak E-field (MV/m)	33	32	47
Peak B-field (mT)	56	61	71
R/Q (Ω)	287	915	318
Nearest mode (MHz)	584	370	575

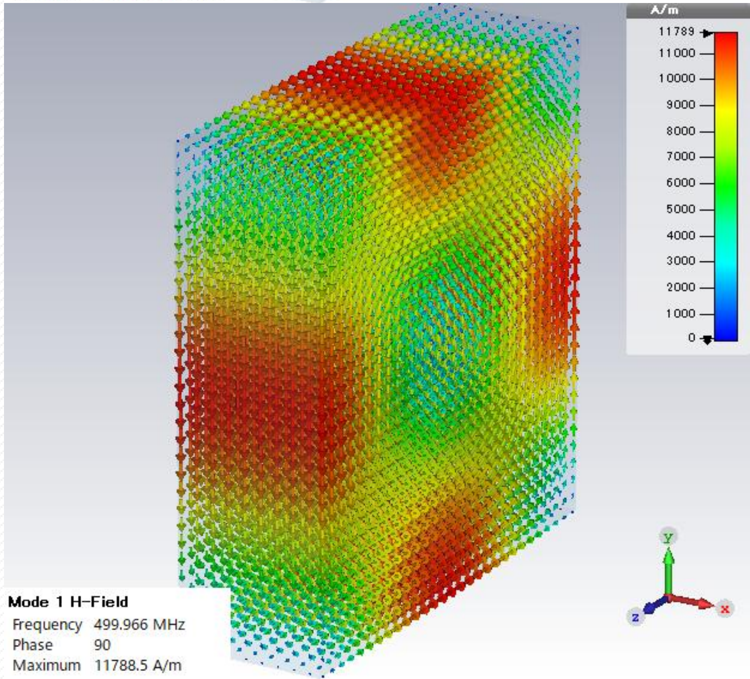
RFD and DQW were chosen for LHC CC

Analytical solution of EM fields

- Consider EM fields in a rectangular cavity
- Obtain analytical solutions of the TE011 mode
- Calculate its kick voltage



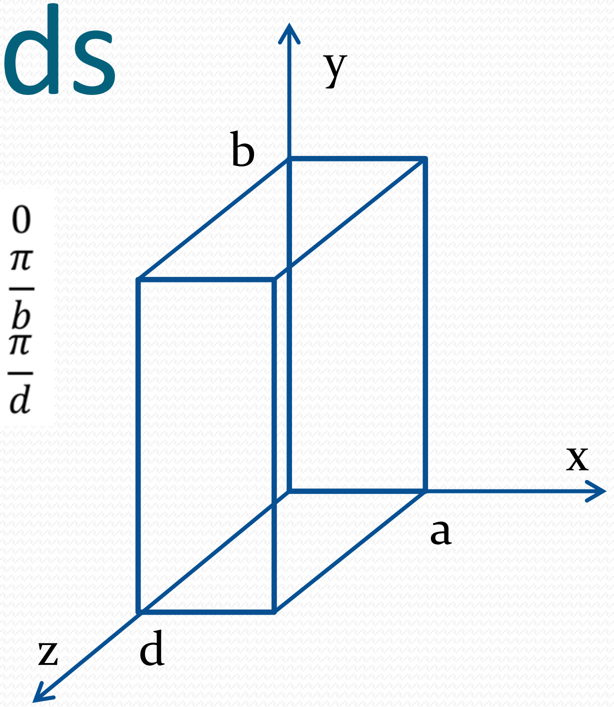




Electro-magnetic fields

$$E_x = iE_0 \frac{kk_y}{k_c^2} \cos k_x x \sin k_y y \sin k_z z$$
$$E_y = -iE_0 \frac{kk_x}{k_c^2} \sin k_x x \cos k_y y \sin k_z z$$
$$E_z = 0$$
$$H_x = -\frac{E_0}{c\mu} \frac{k_x k_z}{k_c^2} \sin k_x x \cos k_y y \cos k_z z$$
$$H_y = -\frac{E_0}{c\mu} \frac{k_y k_z}{k_c^2} \cos k_x x \sin k_y y \cos k_z z$$
$$H_z = \frac{E_0}{c\mu} \cos k_x x \cos k_y y \sin k_z z$$

$$k_x = 0$$
$$k_y = \frac{\pi}{b}$$
$$k_z = \frac{\pi}{d}$$



$$CB_x = -E_0 \frac{k_x k_z}{k_c^2} \sin k_x x \cos k_y y \cos k_z z$$
$$CB_y = -E_0 \frac{k_y k_z}{k_c^2} \cos k_x x \sin k_y y \cos k_z z$$
$$CB_z = E_0 \cos k_x x \cos k_y y \sin k_z z$$

Integration of analytical solution

- Calculate kick voltage for the TE mode in a rectangular cavity
  - Electromagnetic fields are given
  - A particle passes at x=a/2, y=b/2
  - Time factor $e^{i\omega t}$

$$E_x = iE_0 \frac{kk_y}{k_c^2} \sin k_y y \sin k_z z$$

$$CB_y = -E_0 \frac{k_y k_z}{k_c^2} \sin k_y y \cos k_z z$$

$$E_x = iE_0 \frac{kk_y}{k_c^2} \sin k_z z$$

$$CB_y = -E_0 \frac{k_y k_z}{k_c^2} \cos k_z z$$

$$E_x = iE_0 \frac{kk_y}{k_c^2} \sin k_z z \quad e^{i\omega t}$$

$$CB_y = -E_0 \frac{k_y k_z}{k_c^2} \cos k_z z \quad e^{i\omega t}$$

Integration of analytical solution

- Calculate kick voltage for the TE mode in a rectangular cavity
  - Real part

$$E_x = -E_0 \frac{k k_y}{k_c^2} \sin k_z z \sin kz$$
$$CB_y = -E_0 \frac{k_y k_z}{k_c^2} \cos k_z z \cos kz$$

  - Imaginary part

$$E_x = E_0 \frac{k k_y}{k_c^2} \sin k_z z \cos kz$$
$$CB_y = -E_0 \frac{k_y k_z}{k_c^2} \cos k_z z \sin kz$$

Integration of analytical solution

- Calculate kick voltage for the TE mode in a rectangular cavity
  - Kick voltage
    - Field integration along beam passage
    - Real part=0
    - Imaginary part=0
    - Kick voltage=0

$$|V_x| = 0$$

$$V_x = \int_0^d (E_x - cB_y) e^{ikz}\, dz$$

$$\int_0^d \sin k_z z \cos kz\, dz = -\frac{k_z}{k_c^2}(1 + \cos kd)$$

$$\int_0^d \cos k_z z \sin kz\, dz = \frac{k}{k_c^2}(1 + \cos kd)$$

$$\int_0^d \sin k_z z \sin kz\, dz = \frac{k_z}{k_c^2} \sin kd$$

$$\int_0^d \cos k_z z \cos kz\, dz = \frac{k}{k_c^2} \sin kd$$

$$ReV_x = Re \int_0^d (E_x - cB_y) e^{ikz}\, dz = 0$$

$$ImV_x = Im \int_0^d (E_x - cB_y) e^{ikz}\, dz = 0$$