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Cryogenic Instrumentation

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Instrumentation and Measurement [1]

Some Definitions:

- Readability / Resolution: It is the degree to which a change can be theoretically detected.
  - *Resolution of a 0-5V voltmeter with 4-bits A/D converter is* $^{(5-0)}/_{2^4} = 0.3125\,V$. *So, we can not expect to get a 0.4V reading from it. A ruler with a minimum graduation of 1.0 mm can only measure lengths in increments of 1.0 mm (i.e. it can not measure 1.5 mm). Here, 1.0 mm is the resolution.*

- Sensitivity: It is the smallest absolute amount of change that can be detected by a sensor. This is relevant when measurements are made in units other than the quantity being measured. Sensitivity is the local slope of the input (e.g. bar) vs output (e.g. mV) curve, at that input value (for a linear sensor, that slope is independent on the input value).
  - *A thermocouple will have a sensitivity (mV/K), but a thermometer will have a resolution (e.g. 0.1^{o}K).*

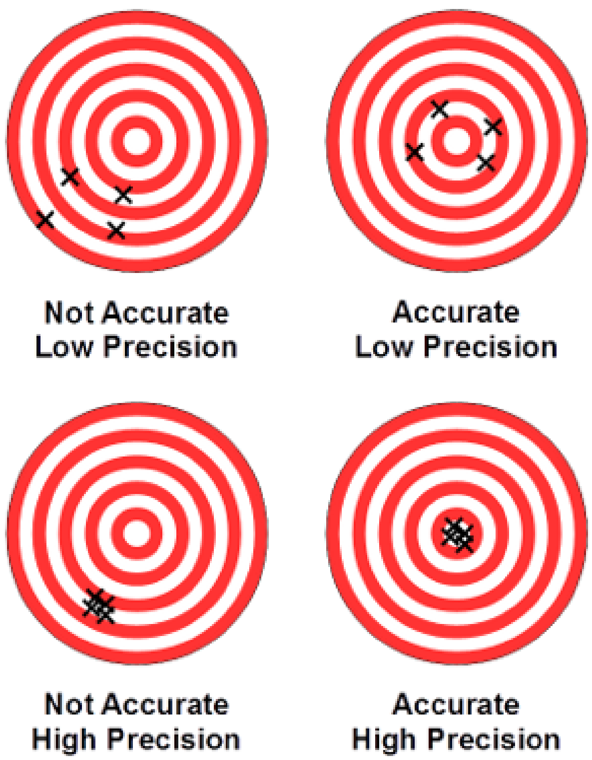
Instrumentation and Measurement [2]

Some Definitions:

- Accuracy: It indicates the deviation of the reading from a know (actual) input. It is frequently expressed as a percentage of full-scale reading.
  - *A 10 bar pressure gage having a full-scale (FS) accuracy of 1%, would be accurate within $\pm$ 0.1 bar over the entire range of the gage (i.e. a '5.0' bar read-out would mean a pressure anywhere between 4.9 – 5.1 bar)*

- Precision: It indicates the ability to reproduce a certain reading with a given accuracy.

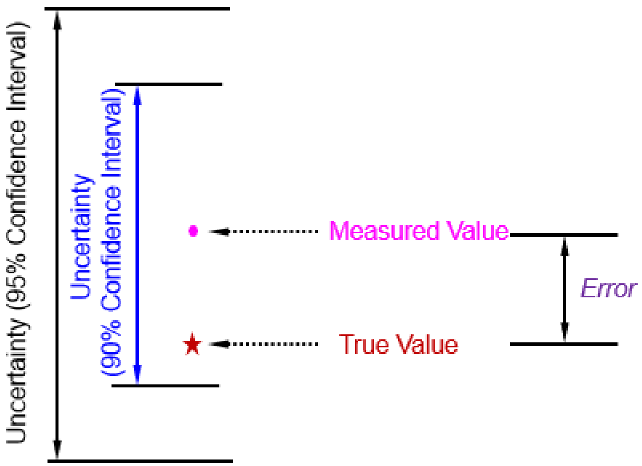
Instrumentation and Measurement [3]



- Accuracy can be improved by calibration. But only up to the precision of the instrument.
- Note that, accuracy is defined as the deviation of the read-back from a known (or actual) value. This deviation is called ***error***.
- In practice, the actual value is the one we are measuring, and in most cases we do not have this known to compare the instrument reading.
- Based on the instrument calibration (and accuracy and precision), we may define a range ($\pm$) within which the instrument will be accurate. This is the ***uncertainty*** in the instrument readings.

Instrumentation and Measurement [3]

Error and Uncertainty:

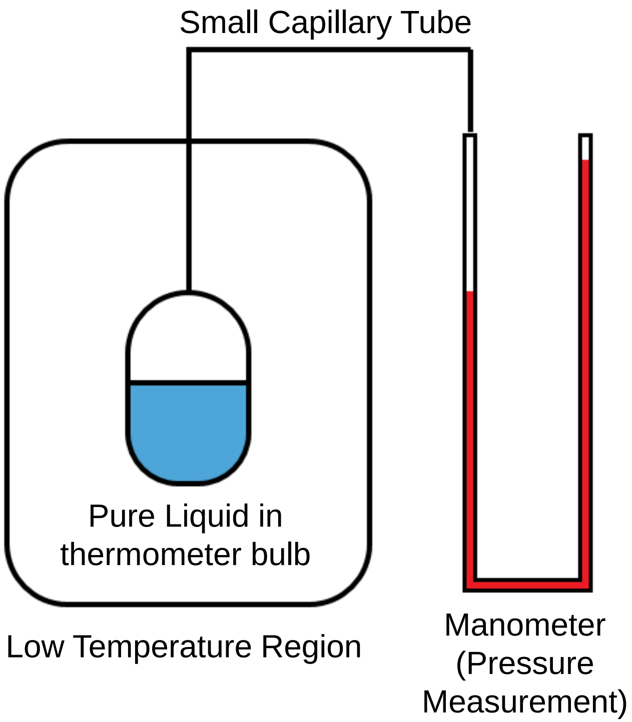


- For instrumentation that rely on multiple different parameters to measure a required quantity, uncertainty of the measurement is dependent on the uncertainty of each of the individual parameter uncertainties.

- For example, a differential pressure flow meter relying on Δp, pressure and temperature measurement to provide a mass flow rate – the uncertainty of the mass flow rate measurement is a function of the uncertainties of each of the differential pressure, flow pressure and temperature measurements (additional uncertainties from other parameters, such as diameter, discharge coefficient needs to be included too).

Temperature Measurement

Vapor Pressure Thermometer [1]



- Vapor (saturation) pressure is a function of the temperature of the liquid in contact with the vapor.
- Measure pressure at room temperature. Pressure is determined by coldest location hence – avoid cold spots and pressure gradient in the capillary.
- Very good sensitivity in the applicable temperature range. For example, a nitrogen vapor-pressure thermometer has a sensitivity of 15 kPa/K to 3 kPa/K in the temperature range from 63K to 80K.

Vapor Pressure Thermometer [2]

- Limited range. Can only be from triple point to critical point of the thermometric fluid. In practice, used for vapor pressures between 5 kPa to 250 kPa (corresponding NBP temperatures of the fluid), unless the fluid reaches triple point/critical point.

- A single thermometric fluid can not be used to measure the entire range from liquid helium temperatures to ambient temperatures.

- *<u>Serves a good reference for calibration</u>*

Vapor Pressure Thermometer [3]

- Thermometric fluid needs to be as pure as possible. Impurities result in departure from the vapor pressure vs. temperature relationship for the pure fluid.
- Vapor pressure for a pure substance may be related to the saturation temperature using –

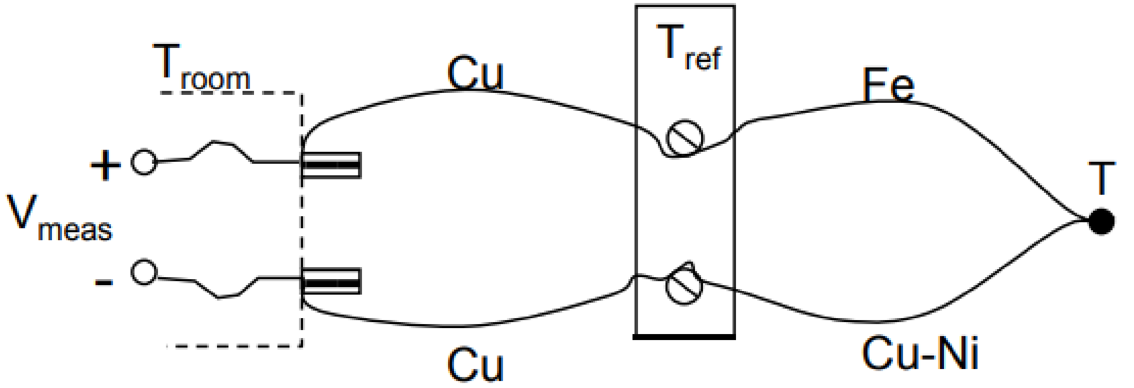
$$ln\left({}^{p}/_{p_0}\right) = C_1 - {}^{C_2}/_{T} - C_3 \ln\left({}^{T}/_{T_0}\right) - C_4 T + C_5 T^2$$

Here, $p_0 = 101.325\ kPa$ and T_0 = NBP temperature of the fluid

	Hydrogen	Neon	Nitrogen	Oxygen
C_1	3.940796	10.618417	13.569758	13.726967
C_2	101.33783	244.96075	930.15333	1076.35667
C_3	0	0	2.3668	1.664512
C_4	-0.0543201	0.0848111	0.0328844	0.0304241
C_5	-1.10563×10^{-4}	9.78350×10^{-4}	1.67138×10^{-4}	1.16981×10^{-4}

Thermocouples [1]

- Consists of two dissimilar electrical conductors forming electrical junctions at differing temperatures (one at the target temperature to be measured, other at a reference temperature). A thermocouple produces a temperature-dependent voltage as a result of the Seebeck effect, and this voltage can be interpreted to measure temperature.
- The Seebeck effect is a phenomenon in which a temperature difference between two dissimilar electrical conductors or semiconductors produces a voltage difference between the two substances.



Thermocouples [2]

- Thermocouples are not commonly used in cryogenic systems – when used, typically in temperature measurement near ambient temperatures.

- In general, lead wires (for measuring EMF) are different material than the thermocouple materials, meaning two new thermocouples will be created in that junction (where thermocouple wires and lead wires are connected). This junction is typically called a '*Cold Junction*'.

- Measured EMF needs to be compensated for '*Cold Junction*'.

- For cold junction compensation, the reference temperature needs to be known accurately (a saturated liquid bath with known NBP is used).

- The electro-motive force (e) generated is then a polynomial function of

$$t = T - T_{reference}$$

$$e = a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4$$

Thermocouples [3]

- The output EMF is quite small – in the order of millivolts (mV). This can be alleviated by using several thermocouples in series, known as a 'thermopile'. EMF is additive – for n thermocouples EMF can be amplified by n times.

- Different types of material combinations are possible. Type T (Copper-Constantan) and Type K (Chromel-Alumel) are commonly used in cryogenic systems.

Thermocouple	Measurement Range	Standard Limits of Error
Type T	-200 to 350 ^oC	2.2 ^oC or 0.75% whichever is greater
Type K	-200 to 1100 ^oC	1.0 ^oC or 0.75% whichever is greater

Thermocouples [4]

- Long term stability and precision in the low temperatures range (below 0ºC) is not good.

- For cryogenic temperature measurement (say liquid nitrogen in a vessel), there will be conduction heat in-leak from the ambient via the thermocouple wires. This heat in-leak will tend to warm the thermocouple bead above the temperature of the mounting surface.

- This can be mitigated by placing the thermocouple wiring (with electrical insulation) at an isothermal region at the low temperature (wrap around the vessel).

- Special design considerations are still required to minimize heat in-leak to system (discussed later).

Resistance Thermometers [1]

- Also known as Resistance Temperature Detectors (RTDs).

- Consists of some type of resistive element, which is exposed to the temperature to be measured. The temperature is indicated through a measurement of the change in resistance of the element.

- Various types of materials may be used as the resistive element.

- The linear temperature coefficient of resistance, α is given by –

$$\alpha = \frac{R_2 - R_1}{R_1(T_2 - T_1)}$$

- Here, R_1 and R_2 are the resistances of the element at temperatures T_1 and T_2. The above relationship is generally applied over a narrow temperature range (over which the R-T relationship is linear)

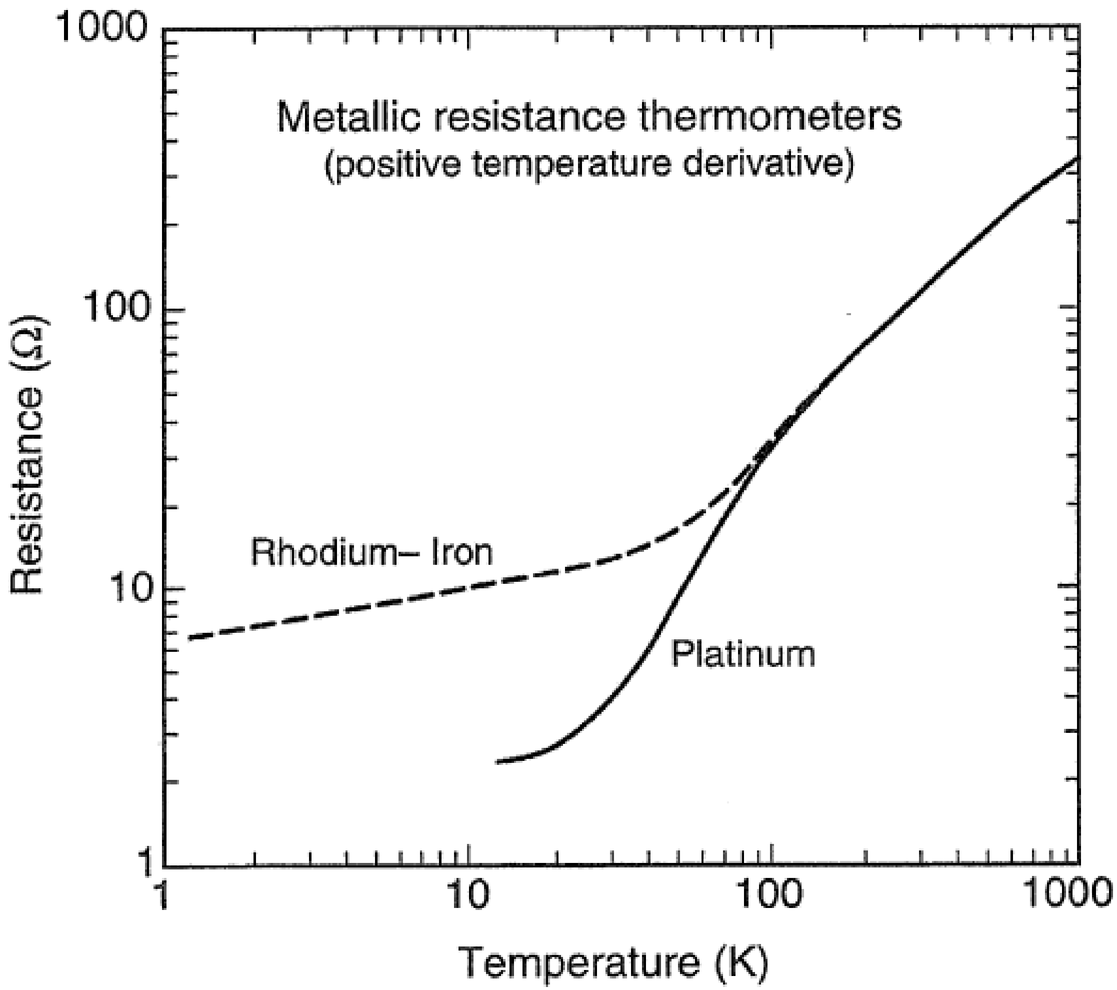
Resistance Thermometers [2]

- Can be differentiated to the following two types –

Positive Temperature Coefficient (PTC) Thermometers:

  - Also known as Metallic resistance thermometer.
  - Have a temperature derivative $\left(\frac{dR}{dT}\right)$ that is positive.
  - Highly reproducible (high precision), and interchangeable (i.e. can be interchanged with another sensor without significant change in calibration).
  - Becomes less sensitive at lower temperatures.

Resistance Thermometers [3]



Resistance Thermometers [4]

Positive Temperature Coefficient (PTC) Thermometers:

- Commonly used materials for cryogenic temperature measurement are – Platinum, Rhodium-Iron alloy, copper, indium etc.
- Platinum resistance thermometers are highly precise for measurements above ~ 77 K.
- In the lower temperature range (< 20 K), Rhodium-Iron resistance thermometers are more sensitive than platinum sensors.
- Typical industrial PTC thermometer R-T relationship is given by Callender-van Dusen Equation –

$$\frac{R}{R_0} = 1 + AT + BT^2 + CT^3(T - 100)$$

- Here, R_0 is the resistance at T = 0ºC. The constants A, B and C are found by calibration at three standard temperatures.
- Important to mount the resistance thermometer in such a manner that thermal and mechanical strains are eliminated. Strain causes change in electrical resistance in addition to the temperature effect.

RTD – Example Problem

A platinum resistance thermometer yields a resistance reading of 38.6 ohms at a certain temperature. If the electrical resistance at 0°C is 100 ohms, determine the corresponding temperature indication and the sensitivity of the thermometer.

Here, $A = 3.946 \times 10^{-3} {}^{\circ}C^{-1}$, $\mathrm{B} = -1.108 \times 10^{-6} {}^{\circ}C^{-2}$ and $\mathrm{C} = 3.33 \times 10^{-12} {}^{\circ}C^{-4}$

Solution:

For a platinum resistance thermometer,

$$\frac{R}{R_0} = 1 + AT + BT^2 + CT^3(T - 100)$$

$$\frac{38.6}{100} = 1 + 3.946 \times 10^{-3}T - 1.108 \times 10^{-6}T^2 + 3.33 \times 10^{-12}T^3(T - 100)$$

Use an iterative method / polynomial solver / Excel GoalSeek function etc. –

Real solution: $T = -1079.15^{o}C$ (un-physical solution)

$T = -149.99^{o}C$ (actual solution)

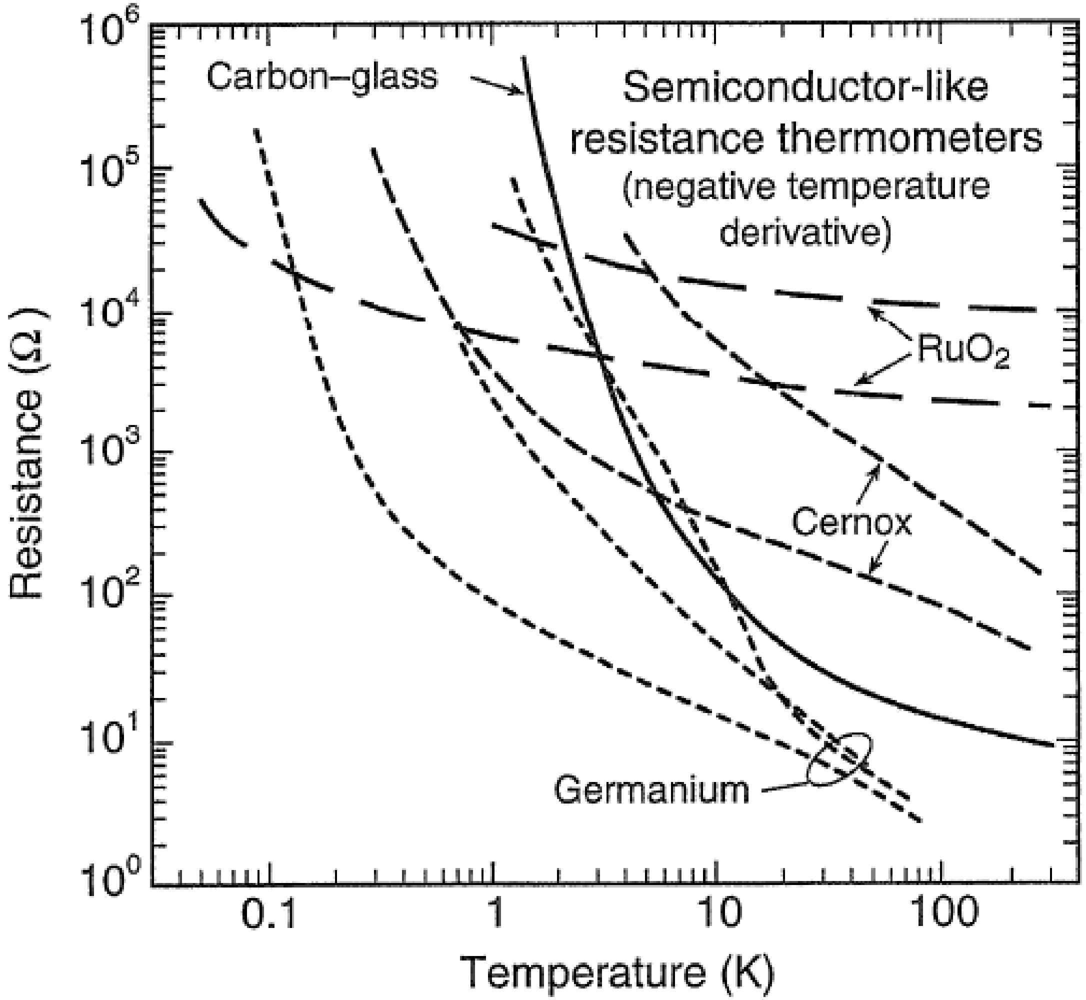
Sensitivity: $S_0 = \frac{\partial R_e}{\partial T} = R_0[A + 2BT + CT^2(4T - 300)] = 0.4211\ \Omega/{}^{\circ}C$

Resistance Thermometers [5]

Negative Temperature Coefficient (NTC) Thermometers:

- Also known as Semiconductor-like resistance thermometers.
- Electrical conductivity of semiconductors are temperature dependent. Conductivity depends on impurity doping level.
- Have a temperature derivative $\left(\frac{dR}{dT}\right)$ that is negative.
- Not interchangeable (i.e. can not be interchanged with another sensor without significant change in calibration).
- Usually requires individual calibration.

Resistance Thermometers [6]



Resistance Thermometers [7]

Negative Temperature Coefficient (NTC) Thermometers:

- In contrast to metallic (PTC) resistance thermometer, they have good sensitivity ($S = \frac{\partial R}{\partial T}$) at low temperatures, but become less sensitive at higher temperatures.

- Commonly used materials for cryogenic temperature measurement are – Zirconium-Oxynitride (Cernox™), Germanium, Carbon-Glass, Ruthenium Oxide etc.

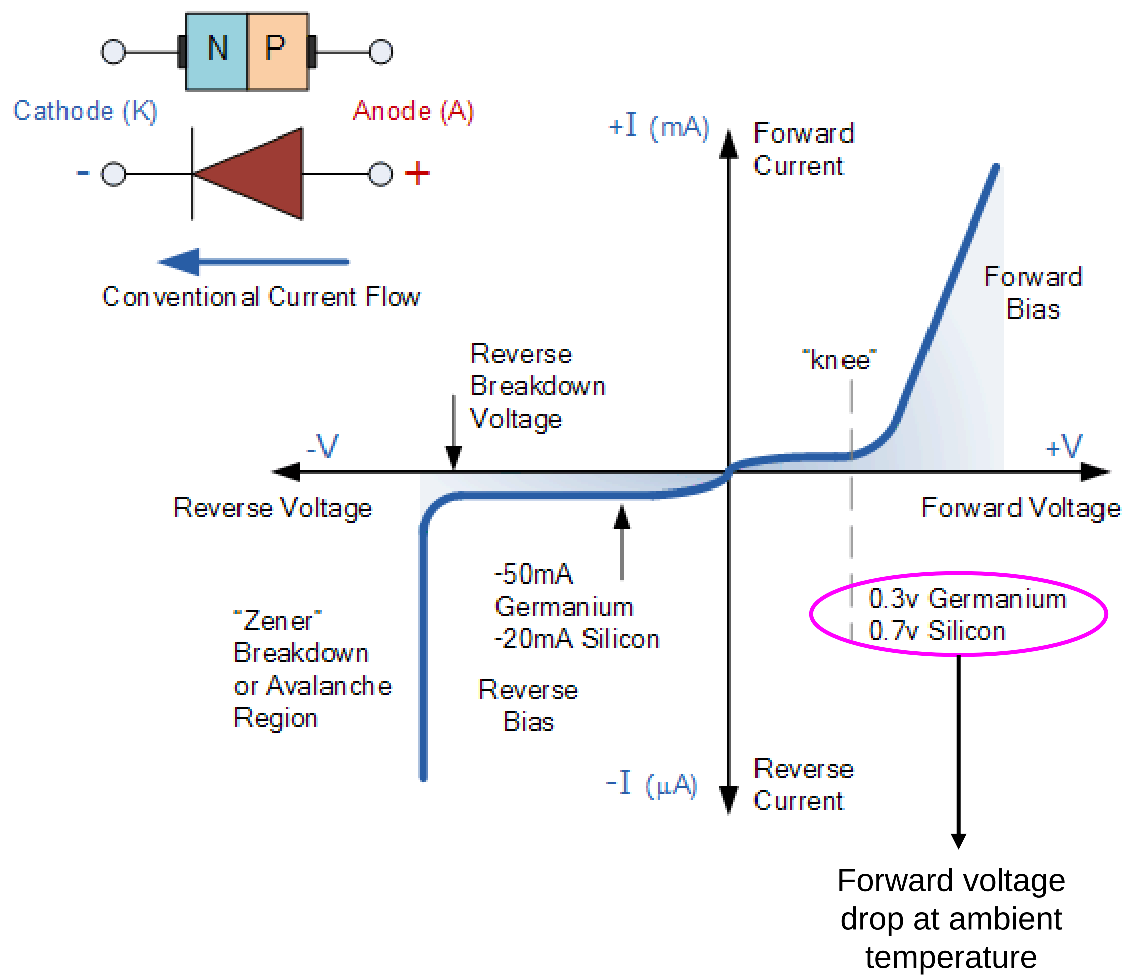
- Cernox™ can be used for measurements in the range of 0.3 – 325 K.

- Germanium can be used for measurements in the range of 0.05 – 100 K.

Diode Thermometers [1]

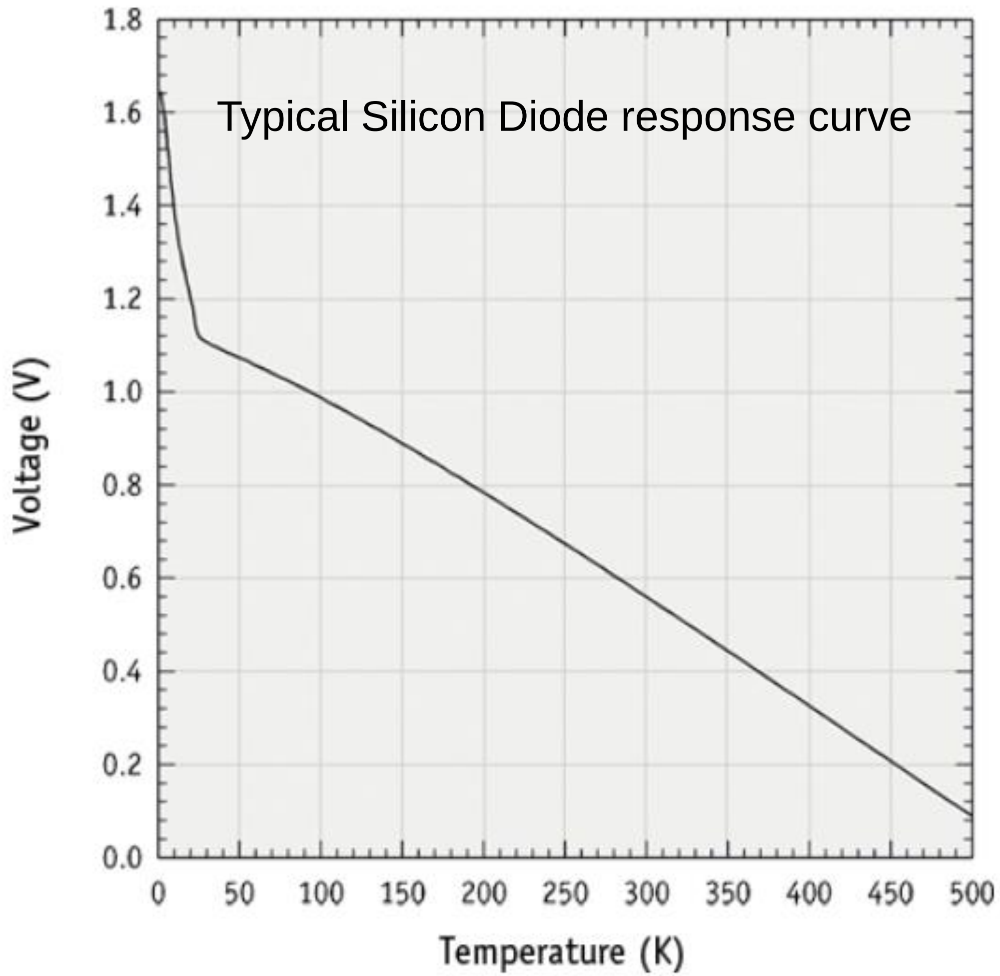
- Forward voltage drop in a p-n junction diode has a rather strong negative temperature dependence.
- Diode thermometry is based on the temperature dependence of the forward voltage drop in a p-n junction biased at a constant current, typically 10 μA.
- Because the voltage signal is relatively large, between 0.1 V and 6 V, diodes are easy to use and instrumentation is straightforward.
- Very good choice for general-purpose cryogenic use.

Diode Thermometers [2]

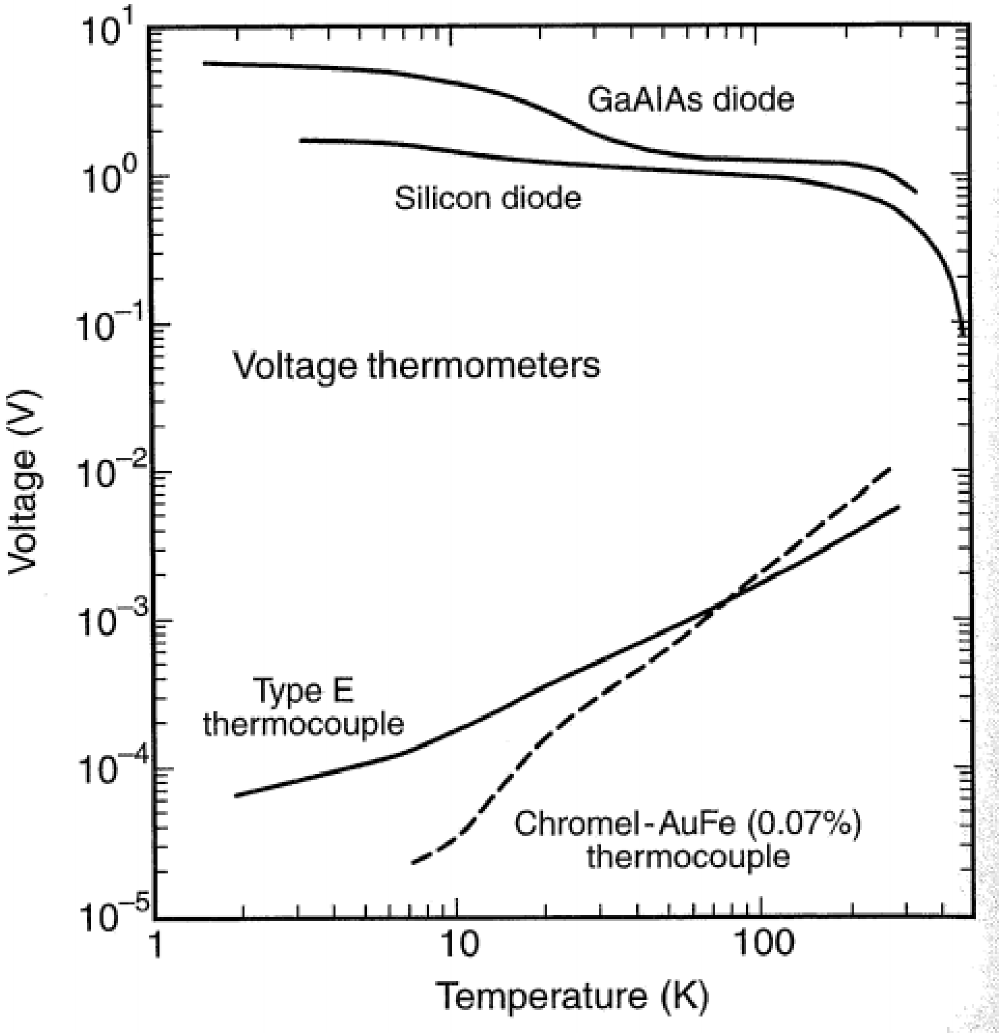


Diode Thermometers [3]

- Highly sensitive in the low temperature region.
- Silicon diodes are easy and inexpensive to instrument, and are used in a wide variety of cryogenic applications. Such as cryo-coolers, laboratory cryogenics, cryo-gas production, and space satellites.



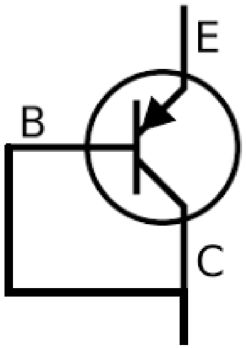
Diode Thermometers [4]



Diode Thermometers [5]

- In practice, a diode connected bipolar junction transistor (BJT) rather than a standard true diode is recommended. This is because BJTs have consistent temperature coefficient which results in smaller errors over temperature.

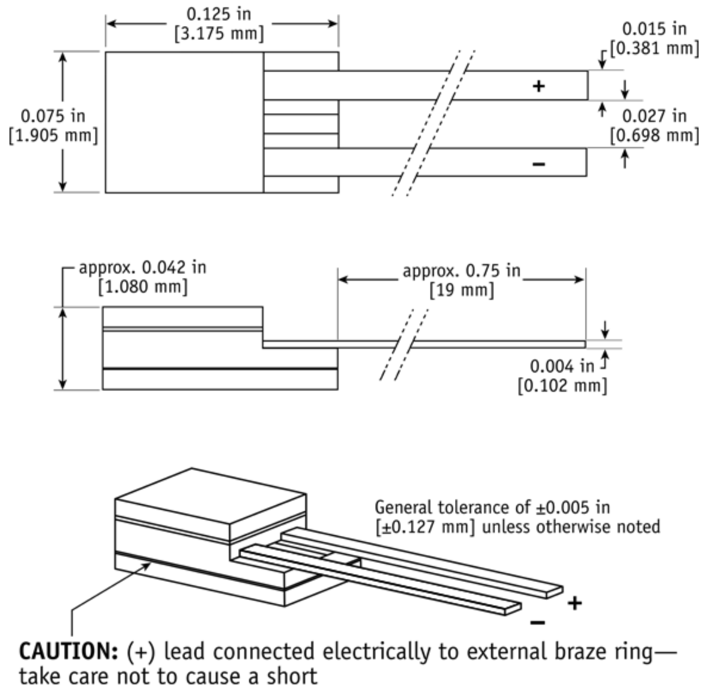
- *A diode-connected transistor is made by connecting the base and collector of a BJT*

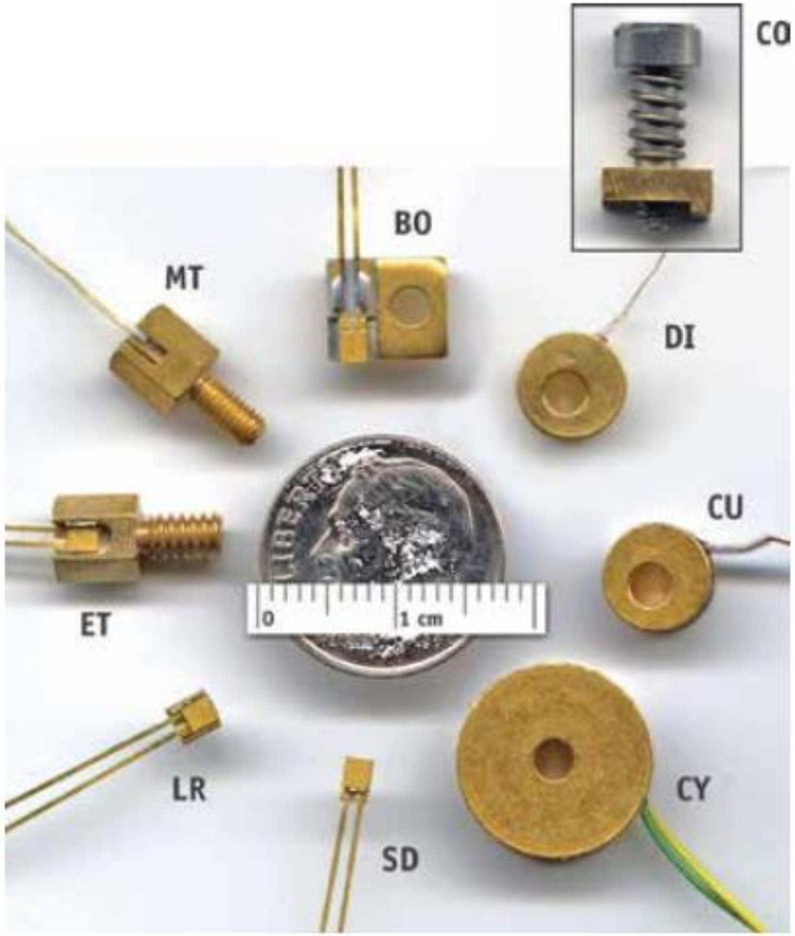


- Typical temperature range for Silicon diodes are 1.4 K – 450 K.

- The sensors are interchangeable (they follow a standard curve) and are available in robust mounting packages and probes.

Diode Thermometers [6]





Silicon diode package with sapphire base, alumina body and lid. Molybdenum/manganese metallization on base and lid top with nickel and gold plating. Gold tin solder as hermetic lid seal.

Image Courtesy: Lakeshore Cryotronics

Summary of Cryogenic Thermometry

Sensor Type	Temperature Range	Accuracy (± value)	Reproducibility (± value)	Interchange-ability	Magnetic Field Use	Best Use	Cost
Platinum resistance thermometer	77–800 K With impurity correction: 20–77 K	Without individual calibration: 0.6 K at 70 K 0.2 K at 300 K With individual calibration: 20 mK at 70 K 35 mK at 300 K	10 mK from 77 K to 305 K	Yes	Recommended above 70 K; error < 0.1% with standard correction	Measurements above 77 K Excellent reproducibility interchangeability, low magnetic field error Many shapes and sizes available	Low without calibration High with individual calibration
Zirconium–oxynitride resistance thermometer (Cernox™)	0.3–325 K	Must be individually calibrated 5 mK at 4.2 K < 0.1% at > 10 K	3 mK at 4.2 K	No	Recommended	One of the best sensors for use in magnetic fields Good sensitivity over a wide temperature range Fast response time as chip	High with individual calibration
Germanium resistance thermometer	0.05–100 K	Must be individually calibrated With individual calibration: 5 mK at < 10 K 0.07% at > 10 K	0.5 mK at 4.2 K	No	Not recommended	Secondary-standard thermometer Excellent reproducibility	High with individual calibration
Silicon diode thermometer	1.4–450 K	Without calibration: 1 K at < 100 K 1% at 100 K to 300 K With individual calibration: 20 mK at 1.4 K–10 K 50 mK at 10 K–330 K	5 mK at 4.2 K 20 mK at 77 K 15 mK at 300 K	Yes	Not recommended below ~60 K	Relatively inexpensive, interchangeable, easily measured output Small size	Medium for low accuracy High with individual calibration

Temperature Measurement Practical Considerations

Installation and Mounting of Thermometers [1]

- Four wire measurements (V+, V-, I+,I-) is generally used for temperature sensors to avoid impact of lead resistance on measurement.
- Wires are in twisted pairs (V+,V-) and (I+,I-) to reduce noise pickup
- Lead wires connect from 300 K to cryogenic temperatures so design must be with small cross sections, and low thermal conductivity.
- 36 gage Manganin wire is a frequent choice (low thermal conductivity integral).
- Using wires that are too fine results in breakage and poor reliability.
- Often lead wires are intercepted at an intermediate temperature to reduce heat in-leak.

Installation and Mounting of Thermometers [2]

- Over constraining the wires is not recommended. Should have room for movement and shrinkage during cool down to avoid breakage.

- Proper anchoring to the target surface is required to measure the surface temperature and not that of the wire due to heat in-leak. Small heat capacities at cryogenic temperatures means small heat leaks can easily impact sensor temperature.

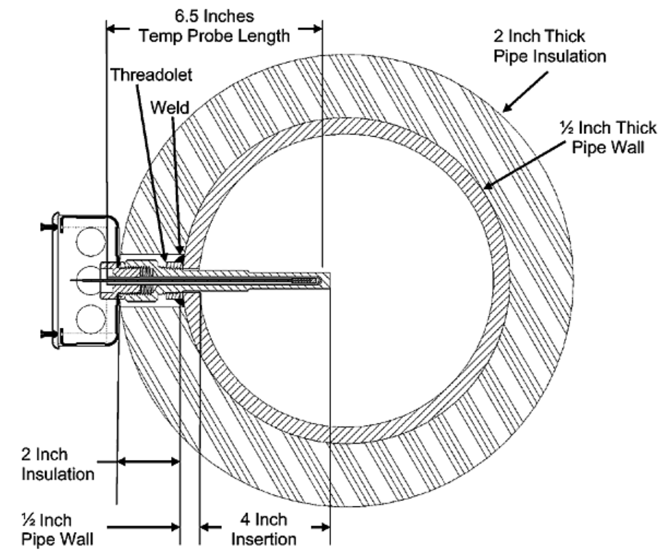
Installation and Mounting of Thermometers [3]

In-Process / Immersion Mounting

- For in-process or immersion type mounting of the thermometer, a thermowell is used. It is a closed-end reentrant tube designed for insertion of a temperature-sensing element, and provided with means for a pressure-tight attachment to a vessel.

- Connection to process may be welded, threaded or flanged.

- Different sensors ranging from thermocouple to PTC, NTC and diodes can be used as the thermometer element.



4" Thermowell in an 8 NPS Schedule 80 Pipe

Installation and Mounting of Thermometers [4]

In-Process / Immersion Mounting

- Often used in large cryostats, tanks and pipes with large cross-section where significant variation of temperature in the radial direction is expected.

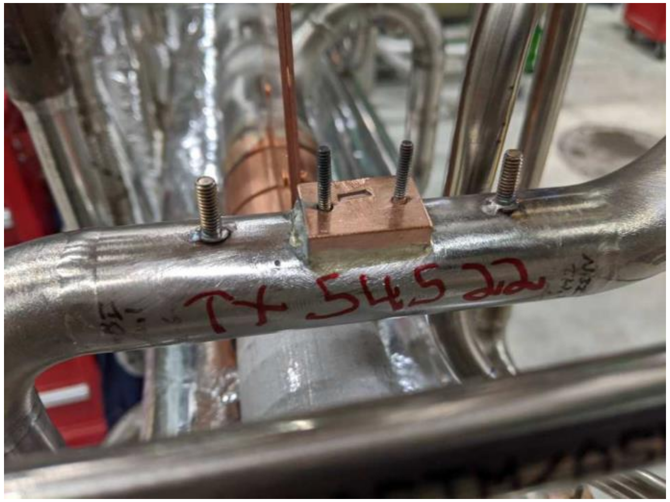
- Design must ensure proper contact between thermowell wall and the sensing element.

- Additional thermal mass induces thermal lag, hence transient response is slow.

Installation and Mounting of Thermometers [5]

Surface Mounting

- Most common for smaller systems, piping etc.
- Often mounted on a block of copper which is brazed to the pipe.
- Sometimes a copper sheet is used instead. It is clamped to the target surface (pipe or vessel outside wall).
- Mounting should provide enough clamping force to reduce thermal contact resistance.
- Softer copper foil is sandwiched between the copper plate and target surface to remove any macro-scale gaps and enhance contact.



Pressure Measurement

General Considerations [1]

- Typically carried out at room temperature (using a capillary tube connected to the process stream).

- Problems with room temperature pressure measurement
  - *Thermo-acoustic Oscillations*
    - *Large temperature gradient along the length of the capillary tube induce pressure pulses (travelling at the local speed of sound). These waves bounce back and forth between the colder (cryogenic) end and warmer end of the capillary tube.*
    - *Perturbs the steady-state stream pressure.*
    - *Induces thermo-acoustic convective flow, carrying heat from ambient to the cold end of the tube.*

  - Time response slow for high-speed transients but for most applications this isn't an issue.

General Considerations [2]

- Thermo-acoustic oscillations can be dampened out using a long capillary tube (viscous damping), or an oscillation damper (deal volume – inertial damping).

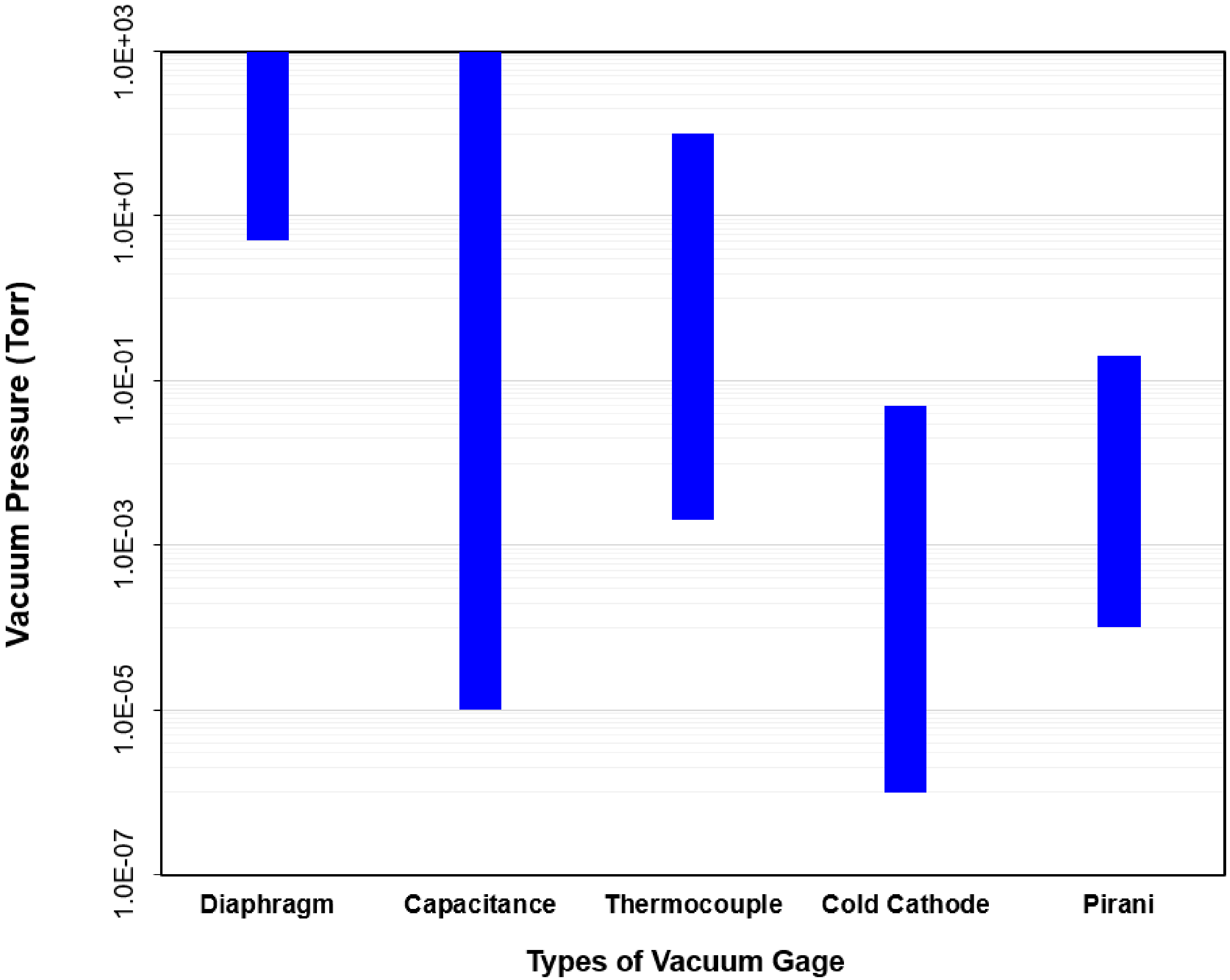
- There are a wide range of 300 K commercial pressure transducers that exist.

- Many are based on capacitive sensors or strain gage bridges mounted on diaphragms that change shape with pressure.

Vacuum Pressure [1]

- Different types of gages can be used depending on range of vacuum pressure.
- Some of these gages such as diaphragm, capacitance etc. measures pressure directly (displacement of a wall due to pressure / change in pressure).
- Some (cold cathode, thermocouple and pirani) measures pressure indirectly. They measure a change in gas property due to change in pressure and converts the change to a measurement (electrical signal).
- A combination of multiple types of gages may be used to cover the entire range of operation.

Vacuum Pressure [2]



Diaphragm Gage [1]

- In the case of the diaphragm vacuum gauge which is capable of absolute pressure measurements, a sealed and evacuated vacuum chamber is separated by a diaphragm from the vacuum pressure to be measured. This serves as the reference quantity.

- With increasing evacuation, the difference between the pressure which is to be measured and the pressure within the reference chamber becomes less, causing the diaphragm flex.

- This flexure may be transferred by mechanical means like a lever, for example, to a pointer and scale, or electrically by means of a strain gauge or a bending bar for conversion into an electrical measurement signal.

- The measurement range of such diaphragm vacuum gauges extends from 1 mbar (0.75 Torr) to over 2000 mbar (1500 Torr).

Capacitance Gage [1]

- The pressure sensitive diaphragm of these capacitive absolute pressure sensors is made of Al_2O_3 ceramics. The term '*capacitive*' means that a plate capacitor is created by the diaphragm with a fixed electrode behind the diaphragm.
- When distance between the two plates of this capacitor changes, a change in capacitance will result. This change, which is proportional to the pressure, is then converted into a corresponding electrical measurement signal.
- With capacitance gauges it is possible to accurately measure pressures from atmospheric pressure to vacuum as low as 10^{-5} Torr.
- Different capacitance gauges having diaphragms of different thickness (and therefore sensitivity) is used to cover the entire range of operation.

A capacitance gage (right) with the transducer and read-out device.

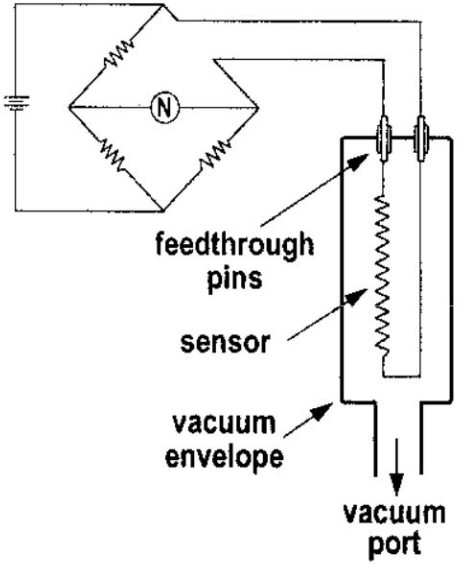
Pirani Gage [1]

- Also known as Thermal Conductivity Gage.

- This measurement principle utilizes the thermal conductivity of gases for the purpose of pressure measurements in the range from 10^{-4} Torr to 0.5 Torr.

- The filament (typically platinum) within the gauge head forms one arm of a Wheatstone bridge.

- The heating voltage which is applied to the bridge is controlled in such a way, that the filament resistance and thus the *temperature of the filament remains constant* regardless of the quantity of heat given off by the filament.

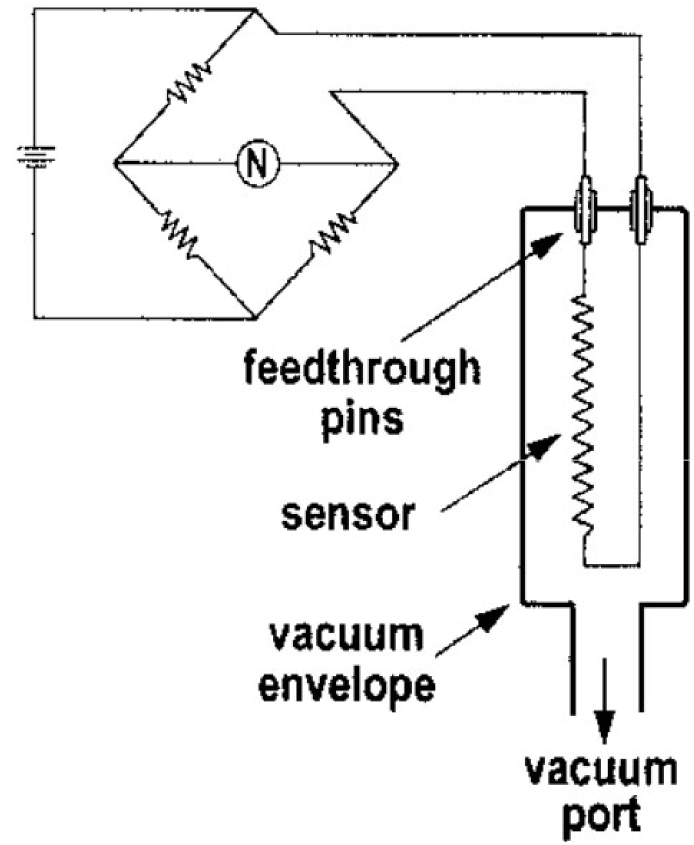


Schematic of a Pirani gage.

Pirani Gage [2]

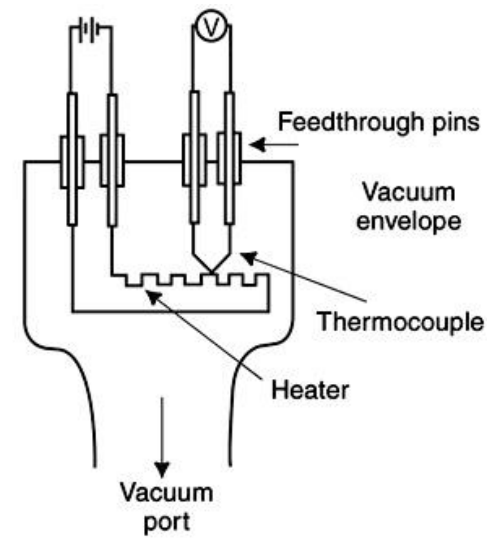
- Since the heat transfer from the filament to the gas increases with increasing pressures, the voltage across the bridge is a measure of the pressure.

- Operation beyond it's high range (~100 Torr) may cause burn-out of the heating filament.



Thermocouple Gage [1]

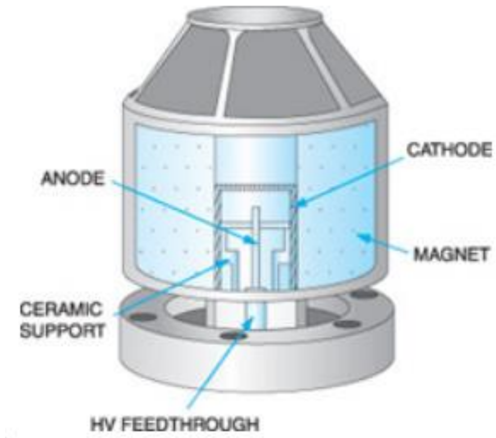
- Another type of Thermal Conductivity Gage.
- Pressure measurements in the range from 0.005 Torr to 100 Torr.
- Constant current is applied to the filament and the temperature of the filament is measured.
- Since the heat transfer from the filament to the gas increases with increasing pressures, the temperature of the heated filament is a measure of the pressure.
- *Note that, this is different from a pirani gage where the temperature of the filament is kept constant by changing the voltage across the filament, while the filament temperature is a variable (measurand) in a T/C gage.*



Schematic of a T/C gage.

Cold Cathode Gage [1]

- A type of Ionization Gage.
- Pressure measurements in the range from 0.05 Torr to 10^{-6} Torr.
- Creates a plasma between two electrodes.
- Ionization current across the electrodes is proportional to the gas (vacuum) pressure.
- In the cold cathode gauges, the ionizing electrons are part of a self-sustaining discharge. At low pressures, this can take minutes and CCGs are usually switched on at high pressure (10^{-2} Torr or higher).
- Once started, the gauge's magnetic field constrain the electrons in helical paths, giving them long path lengths and a high probability of ionizing the residual gas.
- The ions are collected and measured to determine the gas pressure.



Schematic of a CCG.

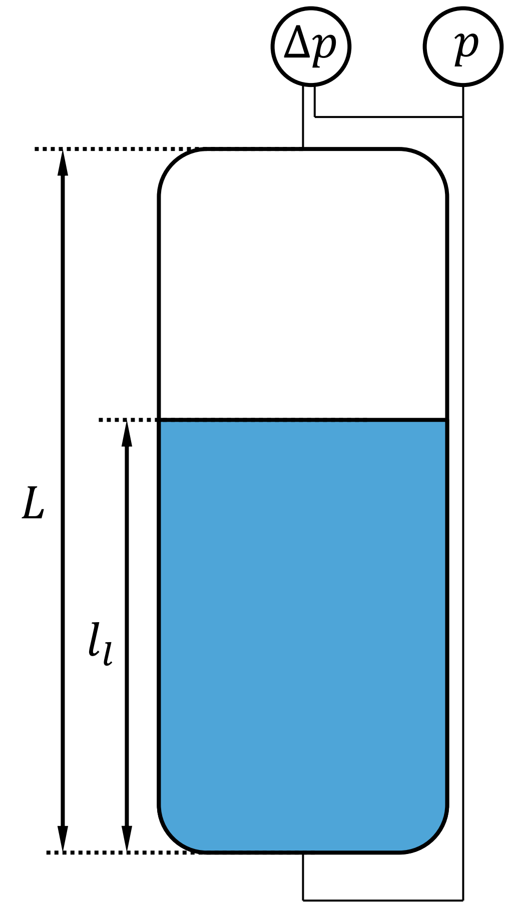
(Liquid) Level Measurement

Hydrostatic (Differential Pressure) Gage [1]

- Often called a differential pressure (Dp) gage.
- Hydrostatic pressure is related to the liquid level (l_l) using –

$$\Delta p = \rho_l l_l g + \rho_v (L - l_l) g$$

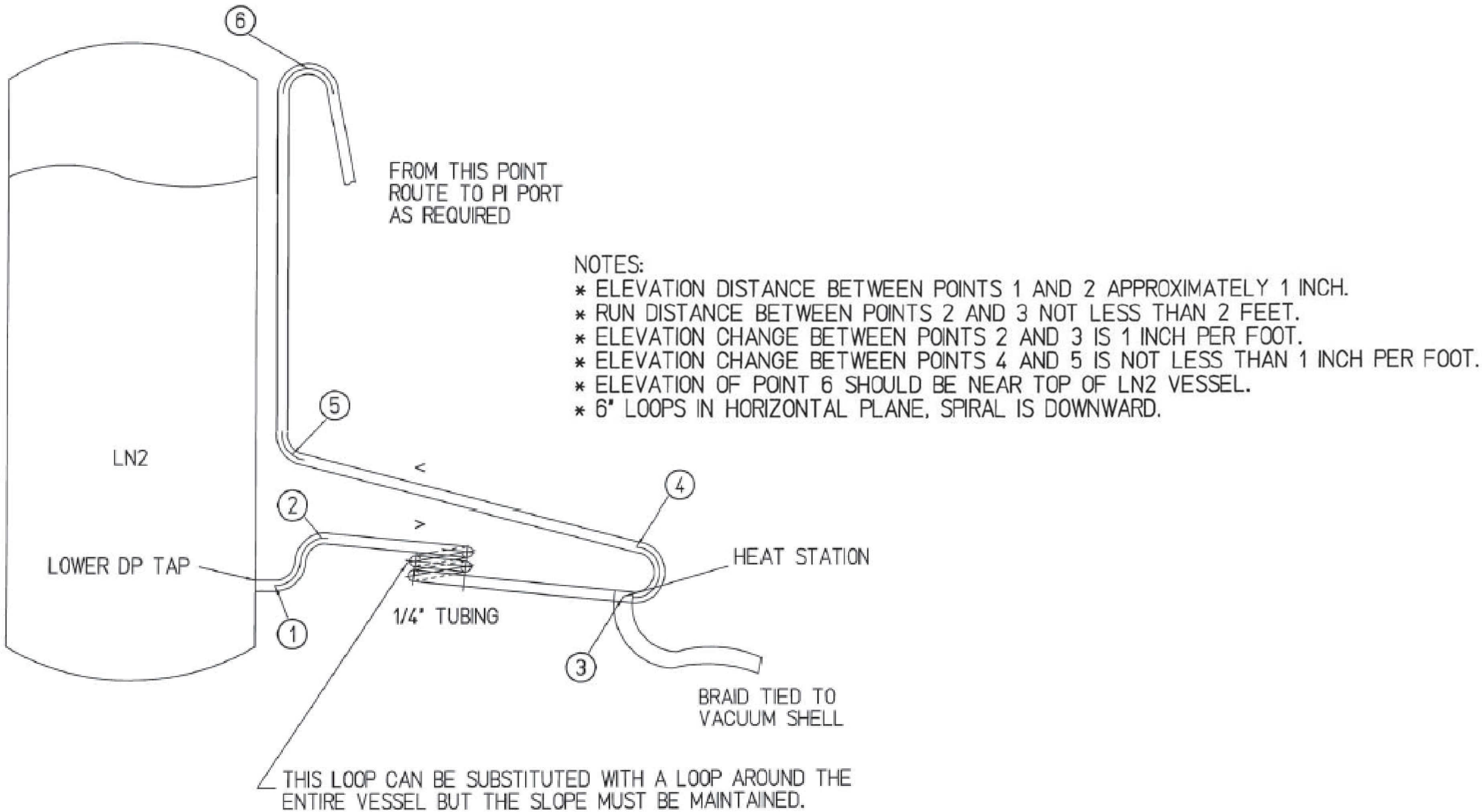
- Knowing the density of the liquid (ρ_l) and vapor (ρ_v), and measuring the hydrostatic pressure difference (Δp), the liquid level can be calculated.
- Accurate measurement of the steady-state saturation pressure (to obtain densities) is required.
- For some fluids, hydrostatic head contribution due to the vapor column is negligible.



Hydrostatic (Differential Pressure) Gage [2]

- For hydrogen and helium, this contribution is significant and should not be neglected.
- Density of liquid helium and hydrogen are significantly lower than other cryogenic fluids. Liquid helium density is approx. 1/8th of that of water.
- Hydrostatic level gage for liquid helium and hydrogen requires a differential pressure gage with high sensitivity.
- Special considerations need to be taken for installation (similar to pressure gage installations). Such as thermo-acoustic oscillations, heat in-leak mitigation etc.
- Blow-down lines are sometimes provided to remove blockage in the capillary tubes due to moisture contamination or vapor locks.

Hydrostatic (Differential Pressure) Gage [3]

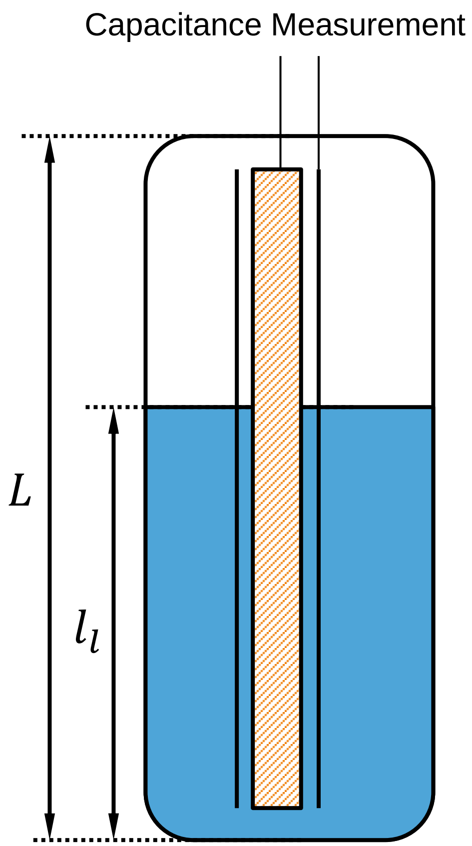


Capacitance Probe / Gage [1]

- Difference in *dielectric constant* between the liquid and vapor phases is the basis of this type of gage.
- A solid rod is placed inside a concentric cylinder and the capacitance of the 'air-gap' is measured. The entire assembly is made long enough to neglect any end effects on the capacitance.
- The total capacitance is given by –

$$C = C_l + C_v = \frac{2\pi[l_l\epsilon_l + (L - l_l)\epsilon_v]\epsilon_0}{ln\left({}^{d_o}/_{d_i}\right)}$$

- Here d_o is the ID of the hollow cylinder and d_i is the OD of the solid rod. ϵ is the dielectric constant. The subscripts l, v and 0 refers to liquid, vapor and free-space (vacuum) respectively.
- Difficult for measuring helium as the difference in dielectric constant between liquid and vapor phases is only 4.0%.



Superconducting Level Gage [1]

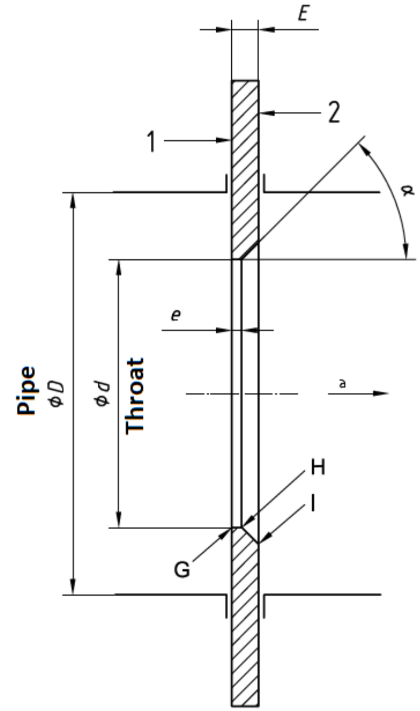
- Operates by measuring the resistance of a superconductive filament contained within a protective tube.
- Used for helium level measurement. Critical transition temperature (temperature below which the material turns superconductive) for most materials is below 10 K.
- An excitation current through the sensor maintains the portion of the filament in helium gas in the normal (resistive) state, while the portion in liquid remains in the superconducting state (zero resistance).
- The resulting voltage along the sensor is proportional to the length of filament above the liquid helium and provides a continuous measure of the helium depth.
- The small amount of heat generated in the probe is dissipated primarily in the helium vapor (resistive part) rather than in the liquid helium (superconducting part).

Mass Flow Rate Measurement

Orifice Plate and Venturi [1]

- Obstructing fluid flow creates a differential pressure across the obstruction. This differential pressure is proportional square of the mass flow rate.
- Mass flow rate can be calculated using

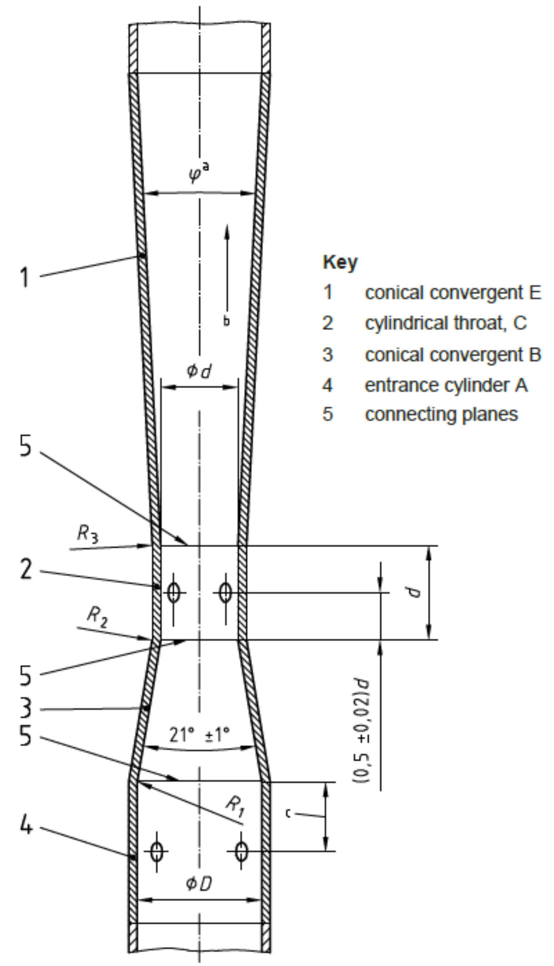
$$\dot{m} = C_d Y A_t \sqrt{\frac{2\rho_1 \Delta p}{1-\beta^4}}$$



- Here,
  - C_d discharge coefficient of the orifice plate.
  - Y expansion factor.
  - A_t cross-sectional area of throat.
  - Δp differential pressure across the orifice plate.
  - ρ_1 upstream density of fluid
  - β ratio of throat to pipe diameter
- Discharge coefficient (C_d) is the ratio of the actual discharge to the theoretical discharge. It depends on pipe diameter, throat diameter, flow rate and flow meter (orifice and venturi) construction.

Orifice Plate and Venturi [2]

- For orifice plates, C_d varies between 0.6-0.65.
- For venturi meters, C_d varies between 0.96-0.99.
- Expansion factor (Y) depends on pipe diameter, throat diameter, pressure ratio across the flow meter (orifice and venture) and isentropic expansion coefficient of fluid at the upstream.
- Y varies between 0.9 – 0.99 (as long as pressure ratio is above 0.75).
- Can not be used for 2-phase flow rate measurement.
- Installation of venturi meter requires certain straight, unperturbed upstream lengths for proper measurement. Flow straighteners are sometimes used.



Orifice Plate and Venturi [3]

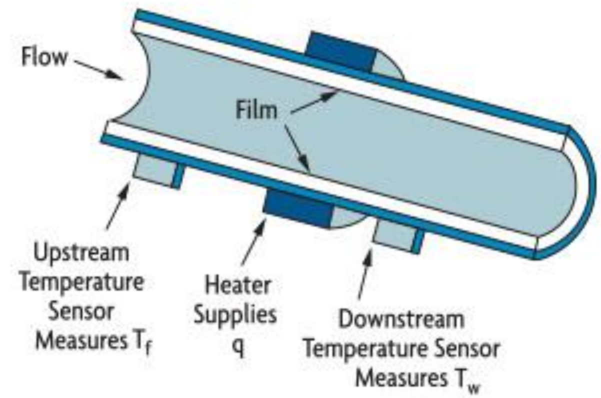
- Accurate knowledge of the discharge coefficient and expansion factor.
- Accurate upstream density (i.e. pressure and temperature) measurement.
- Mounting orientation (vertical / horizontal) needs to be taken in to design consideration. On larger/longer piping hydrostatic head may not be insignificant.
- Most of the pressure drop due to the obstruction in an orifice plate is not recoverable. However, it is compact in shape.
- Most of the pressure drop due to the obstruction in a classical venturi is recoverable. However, it is not compact in shape.
- *When possible mass flow should be measured at warmer / ambient temperature level, as measurement uncertainty (along with heat in-leak and other issues discussed earlier) increases with decreasing temperature.*
- Total uncertainly depends on measurement uncertainty of differential pressure, static pressure, temperature, pipe and throat diameter as well as the uncertainty in estimating the discharge coefficient, and expansion factor.

Thermal Mass Flow Meter [1]

- Thermal mass flow meters are most often used for the measurement of low gas flows.

- They operate either by introducing a known amount of heat into the flowing stream and measuring an associated temperature change or by maintaining a probe at a constant temperature and measuring the energy required to do so.

- The components of a basic thermal mass flow meter include two temperature sensors and an electric heater between them. The heater can protrude into the fluid stream or can be external to the pipe (figure).



$$\dot{m} = \frac{q}{C_p(T_w - T_f)}$$

Thermal Mass Flow Meter [2]

- Introduces direct heating to the possibly cryogenic flow stream (on top of conduction heat in-leak due to the instrumentation lead wiring).
- Not suitable for larger piping where there is significant radial gradient in fluid temperature.
- Not suitable for fluid / operating ranges where specific heat is varies significantly.

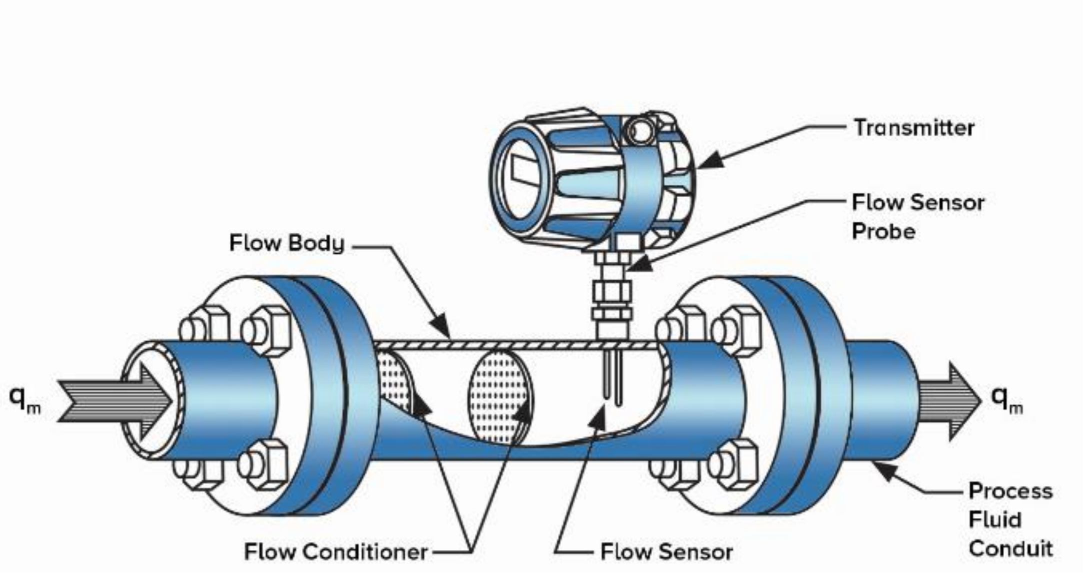


Image Courtesy: Sierra Instruments

Coriolis Mass Flow Meter [1]

Figure 1 Coriolis meter sensor and transmitter

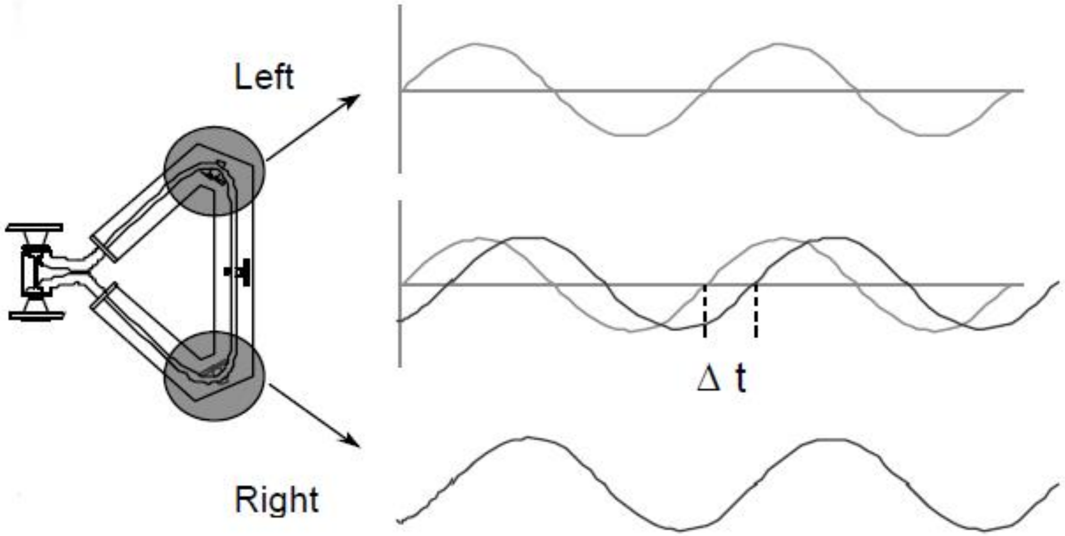


Figure 2 Coriolis flow meter operation

- Two parallel oscillating tubes are used. One with fluid, the other without.
- Flow produces vibrations in the flow tubes that have a phase offset directly related to mass of the tube.
- Measures mass flow rate directly.

Coriolis Mass Flow Meter [2]

- Because mass flow is measured, the measurement is not affected by fluid density (changes). In addition, the relative insensitivity to density allows Coriolis mass flowmeters to be applied in applications where the physical properties of the fluid are not well known.

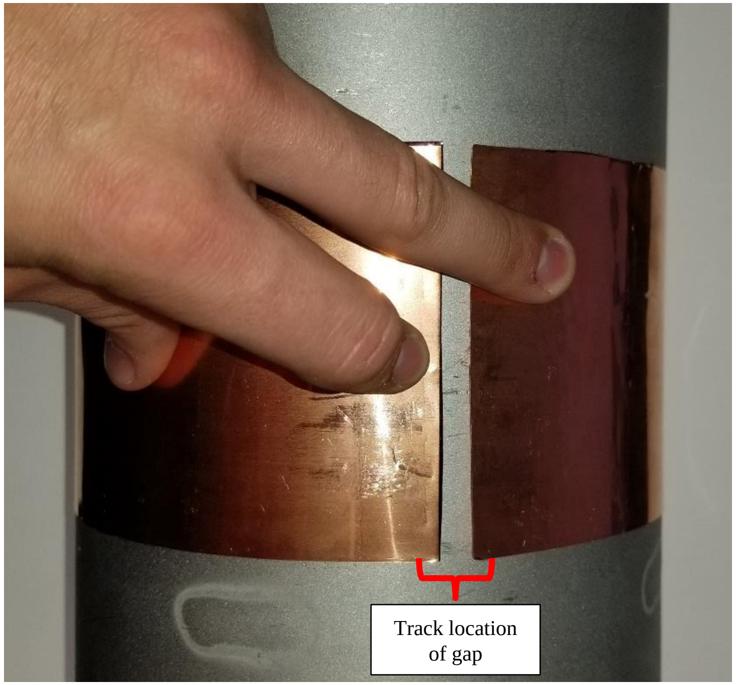
- Have been successfully used to measure mass flow rate of liquid helium at sub-atmospheric pressures (below 2.0 K).

- In gas/vapor applications, large pressure drops across the flowmeter and its associated piping can occur.

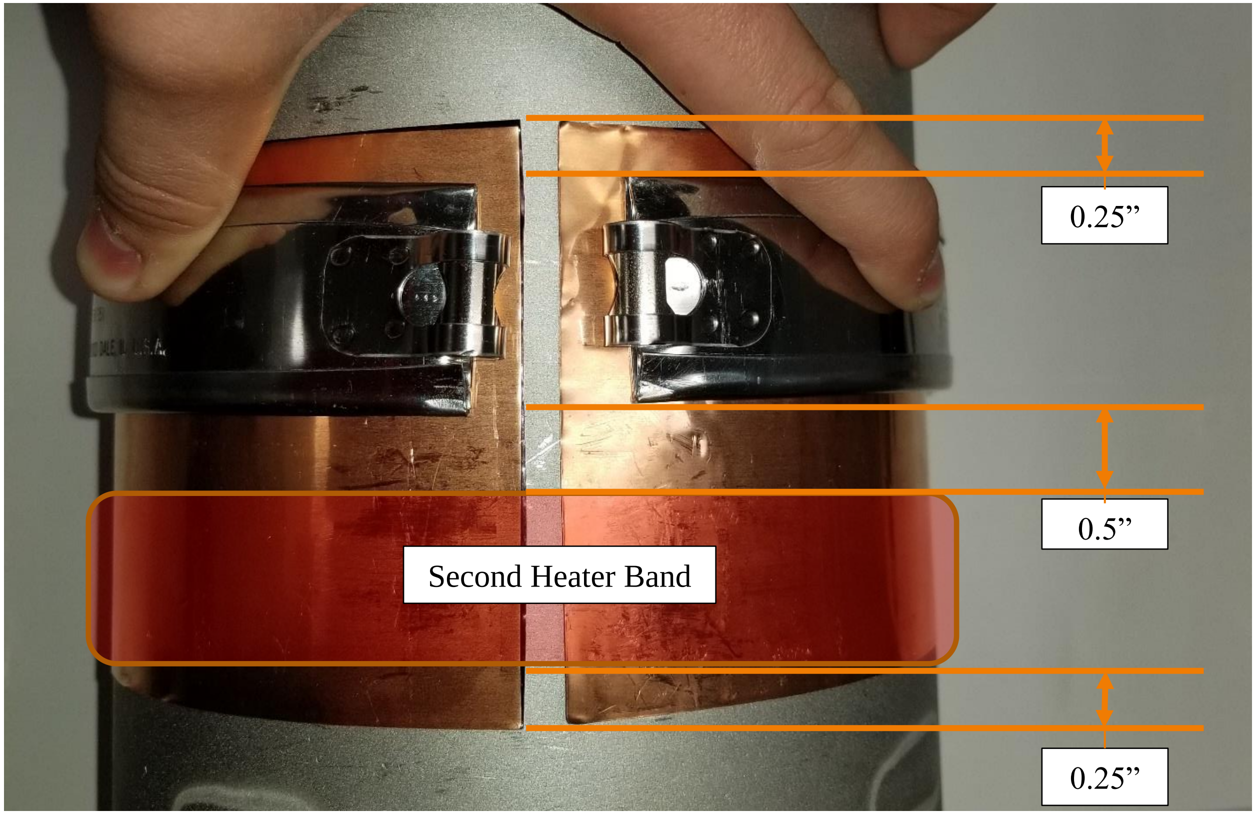
- Very accurate when calibrated properly for cryogenic flow measurement.

Electric Heater Installation

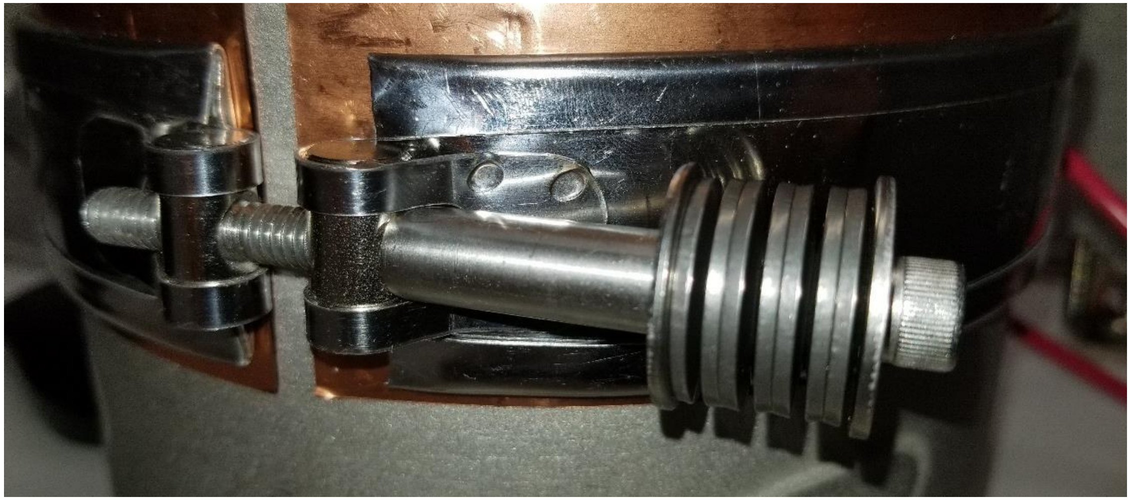
Band Heater Installation [1]



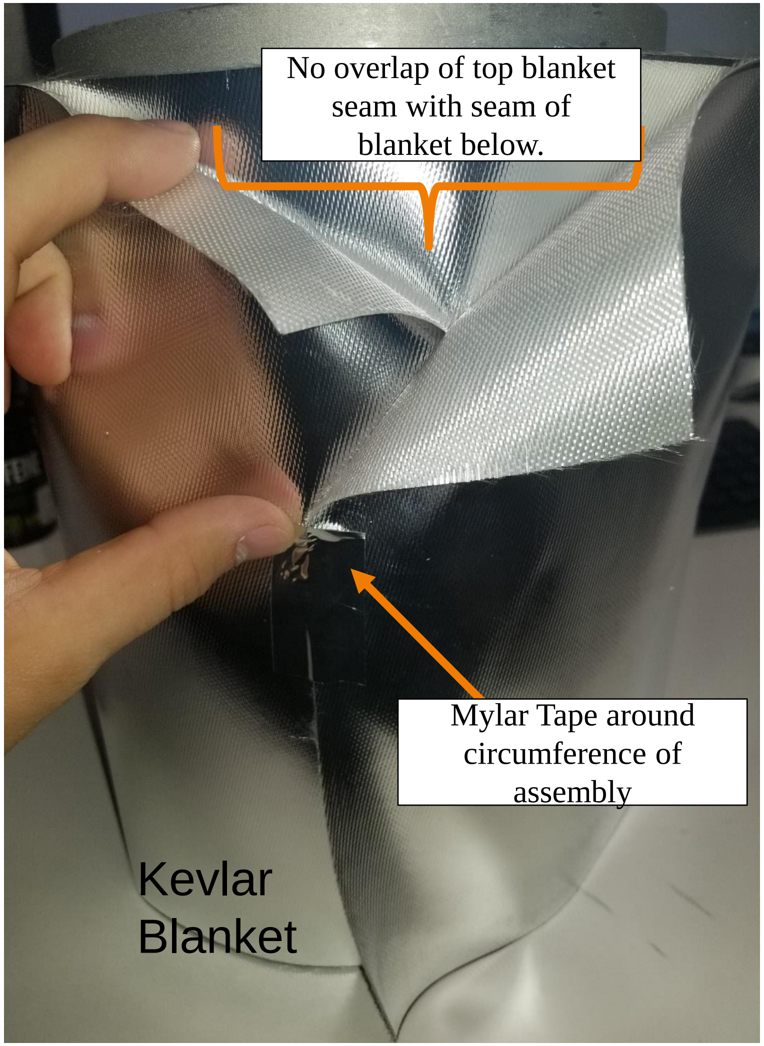
Band Heater Installation [2]



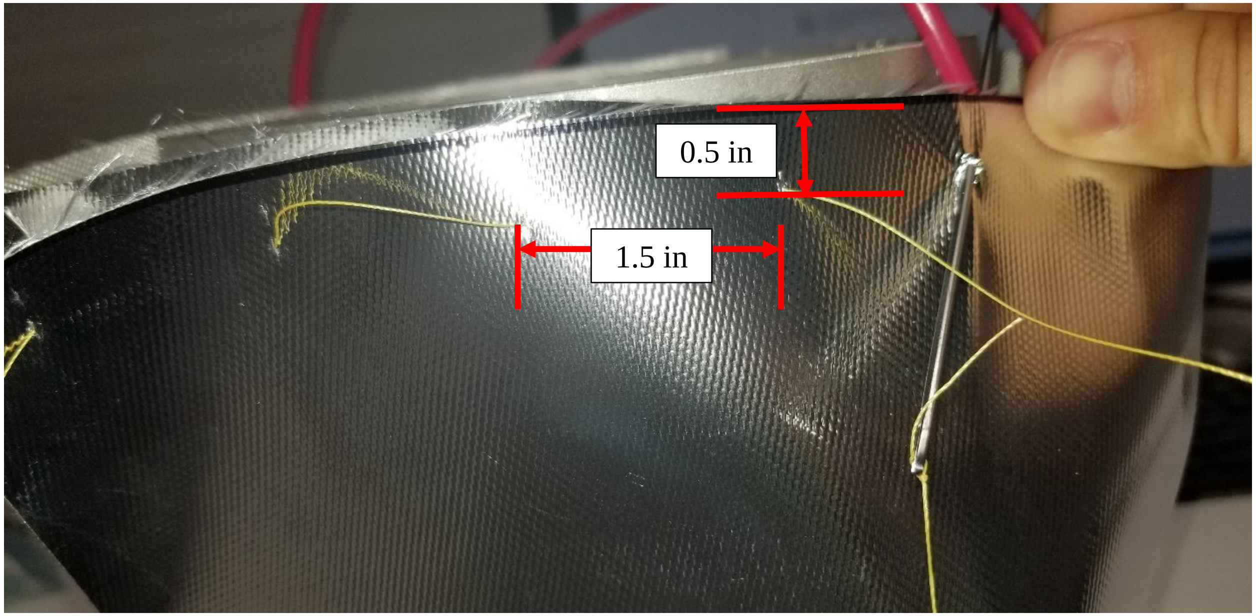
Band Heater Installation [3]

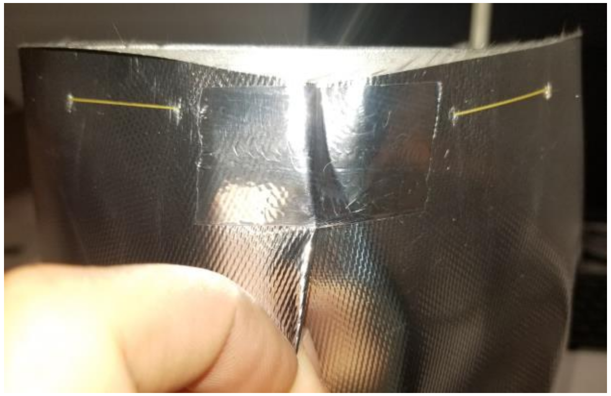


Band Heater Installation [4]



Band Heater Installation [5]





Insertion Heater Installation [1]

- Thermally intercepted by the sat. vapor return piping
- Expansion joints (bellows) to accommodate for thermal contraction
- Typically, finned tubular heating elements are used

Estimating Conduction Heat In-Leak

Conduction Heat Transfer [1]

- At the macroscopic level for an isotropic material,

$$\mathbf{q}''[\mathrm{W/m^2}] = -k[\mathrm{W/m\ K}]\ \nabla T\ [\mathrm{K/m}]$$

- Isotropic – meaning the heat flux vector ($\mathbf{q}''$) has no preferred direction
  - This would be an incorrect assumption for crystals and possibly composites (depending on their construction), in which case the thermal conductivity would be a *tensor*, $k_{i,j}$, rather than a *scalar*
- This is known as Fourier's law of heat conduction, with the operator ∇ being the gradient, $\frac{\partial}{\partial x_i}$, which is why the heat flux is a vector
- This is a 'rate' or 'transport' equation – NOT a conservation law
- Often the thermal conductivity is assumed (treated) to be a constant
- However, in cryogenic applications, in general we must assume that, $k(T)$, is not a constant

Conduction Heat Transfer [2]

- Consider the differential volume, dV, for a solid or non-flowing incompressible liquid with density, ρ, applying the first law in rate form, neglecting K.E. and P.E. terms

$$\frac{dU}{dt} = (q''_x - q''_{x+dx})A_x + other\ directions$$

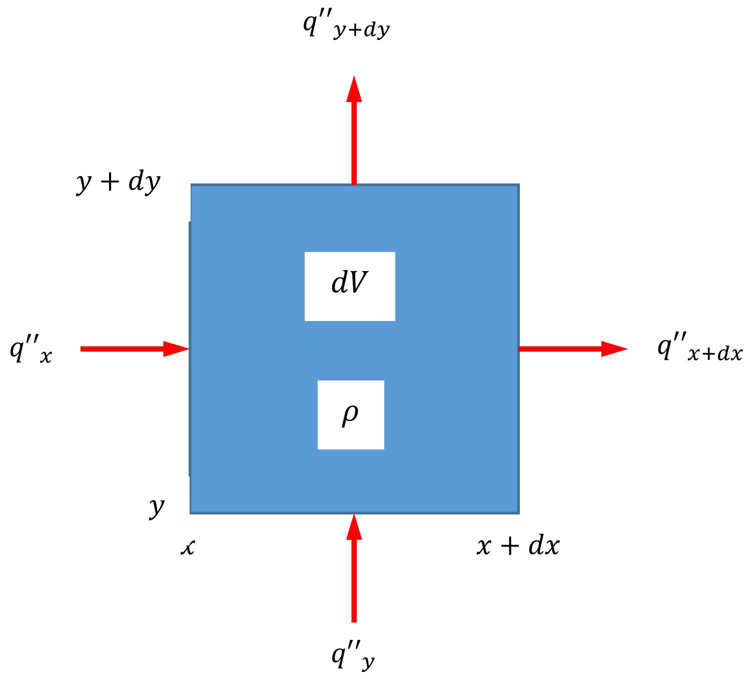
- With

$$A_x = dy\,dz$$

$$\frac{dU}{dt} = \rho\, dV\, \dot{u}$$

$$dV = dx\,dy\,dz$$

$$q''_x = -k\left.\frac{\partial T}{\partial x}\right|_x$$



Conduction Heat Transfer [3]

- So,

$$\rho\,\dot{u} = \frac{\partial}{\partial x}\left(k\,\frac{\partial T}{\partial x}\right) + \cdots = \nabla\cdot(k\,\nabla T)$$

  - Note: This is valid for variable thermal conductivity and specific heat

- With, $du = C_v(T)\,dT$, and, $\alpha = \frac{k}{\rho\,C_v}$, and,
- Defining, $d\mathrm{K} \equiv k(T)\,dT$

$$\frac{1}{\alpha(T)}\dot{\mathrm{K}} = \frac{\partial}{\partial x}\left(\frac{\partial \mathrm{K}}{\partial x}\right) + \cdots = \nabla^2\mathrm{K}$$

- So, the transient conduction equation independent variable has been transformed from the temperature (T) to the *thermal conductivity integral* (K)

Conduction Heat Transfer [4]

- If the specific heat (C_v) and thermal conductivity (k) are constant, this becomes the more familiar transient conduction equation,

$$\frac{1}{\alpha}\dot{T} = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) + \cdots = \nabla^2 T$$

- For steady, one-dimensional conduction, with variable thermal conductivity,

$$\frac{d^2\mathrm{K}}{dx^2} = 0$$

- Recalling that,
  - $q'' = -\frac{d\mathrm{K}}{dx}$, and for the boundary conditions: $(0, \mathrm{K}_1)$, and, (L, K_2),
- So,

$$q = \Delta\mathrm{K}\,\frac{A_c}{L}$$

- Where, $\Delta\mathrm{K} = \mathrm{K}_2 - \mathrm{K}_1 = \int_{T_1}^{T_2} k\, dT$
- And, A_c, is the cross-sectional area normal to, q

Conduction Heat Transfer [5]

- Example: Calculate the heat in-leak through a pair of 4 ft long 12 AWG ETP copper wires supplying power to a heater immersed in liquid helium. Assume wires are in an insulating vacuum and that the heater is off.
  - Per ASTM B258, the diameter of 12 AWG wire is, d = 0.0808 in
  - So, the cross-sectional area is, $A_c = 2 \cdot \frac{1}{4}\pi d^2$ = 0.01026 in^2
  - The length is, L = 48.00 in
  - So, (A_c/L) = 2.136E-4 in^2/in

  - The thermal conductivity of pure metals, like copper, is dependent on what is known as the residual resistivity ratio (RRR)

  - For ETP copper, assume (moderate conservatively) RRR = 100

  - And, for 4 K to 300 K, $\Delta\mathrm{K} = 4920$ W/in
  - So, the heat in-leak is, q = 1.05 W

Conduction Heat Transfer [6]

- This equation (for steady, one dimensional, conduction) can be extended to composite (one-dimensional) problems involving series and parallel heat flow
- Many problems in fluids, heat transfer and electricity, can be characterized as,
  - Flow = Potential (difference) / resistance (to flow)
- In this case the,
  - 'Flow' is the heat transfer (q)
  - 'Potential (difference)' is the temperature difference, which expressed through the thermal conductivity integral ($\Delta \mathrm{K}$), and,
  - 'Resistance' is the inverse of the 'shape' factor (A_c/L)
- For 'series' flow, the 'flow' is constant for each material.
- For 'parallel' flow, the 'potential (difference)' is constant across each material

Conduction Heat Transfer [7]

- Ref. J. Ekin (2006) Experimental Techniques for Low Temperature Measurements, App. A2.1

	$\int_{4K}^{T} \lambda \, dT$ [kW/m]									[W/m]			
	Copper		Copper alloys		Aluminum			Stainless steel	Const-antan	Glass	Polymers		
T[K]	ETP[a]	Phos. deox.	Be/Cu 98 Cu 2 Be	German Silver 60 Cu 25Zn 15 Ni	Common pure 99 Al[a]	Mn/Al 98.5 Al 1.2 Mn plus traces	Mg/Al 96 Al 3.5 Mg plus traces	Average types 303, 304, 316, 347		Average Pyrex™ Quartz Boro-silicate	Teflon™	Perspex™	Nylon™
6	0.80	0.0176	0.0047	0.00196	0.138	0.0275	0.0103	0.00063	0.0024	0.211	0.113	0.118	0.0321
8	1.91	0.0437	0.0113	0.00524	0.342	0.0670	0.025	0.00159	0.0066	0.443	0.262	0.238	0.0807
10	3.32	0.0785	0.0189	0.010	0.607	0.117	0.0443	0.00293	0.0128	0.681	0.44	0.359	0.148
15	8.02	0.208	0.0499	0.030	1.52	0.290	0.112	0.00816	0.0375	1.31	0.985	0.669	0.410
20	14.0	0.395	0.0954	0.0613	2.76	0.534	0.210	0.0163	0.0753	2.00	1.64	1.01	0.823
25	20.8	0.635	0.155	0.102	4.24	0.850	0.338	0.0277	0.124	2.79	2.39	1.44	1.39
30	27.8	0.925	0.229	0.153	5.92	1.23	0.490	0.0424	0.181	3.68	3.23	1.96	2.08
35	34.5	1.26	0.316	0.211	7.73	1.67	0.668	0.0607	0.244	4.71	4.13	2.59	2.90
40	40.6	1.64	0.415	0.275	9.62	2.17	0.770	0.0824	0.312	5.86	5.08	3.30	3.85
50	50.8	2.53	0.650	0.415	13.4	3.30	1.24	0.135	0.457	8.46	7.16	4.95	6.04
60	58.7	3.55	0.930	0.568	17.0	4.55	1.79	0.198	0.612	11.5	9.36	6.83	8.59
70	65.1	4.68	1.25	0.728	20.2	5.89	2.42	0.270	0.775	15.1	11.6	8.85	11.3
76	68.6	5.39	1.46	0.826	22.0	6.72	2.82	0.317	0.875	17.5	13.0	10.1	13.1
80	70.7	5.89	1.60	0.893	23.2	7.28	3.09	0.349	0.943	19.4	13.9	11.0	14.2
90	75.6	7.20	1.99	1.060	25.8	8.71	3.82	0.436	1.11	24.0	16.3	13.2	17.3
100	80.2	8.58	2.40	1.23	28.4	10.2	4.59	0.528	1.28	29.2	18.7	15.5	20.4
120	89.1	11.5	3.30	1.57	33.0	13.2	6.27	0.726	1.62	40.8	23.7	20.0	26.9
140	97.6	14.6	4.32	1.92	37.6	16.2	8.11	0.939	1.97	54.2	28.7	24.7	33.6
160	106	18.0	5.44	2.29	42.0	19.4	10.1	1.17	2.32	69.4	33.8	29.4	40.5
180	114	21.5	6.64	2.66	46.4	22.5	12.2	1.41	2.69	85.8	39.0	34.2	47.5
200	122	25.3	7.91	3.06	50.8	25.7	14.4	1.66	3.06	103.0	44.2	39.0	54.5
250	142	35.3	11.3	4.15	61.8	33.7	20.5	2.34	4.06	150.0	57.2	51.0	72.0
300	162	46.1	15.0	5.32	72.8	41.7	27.1	3.06	5.16	199.0	70.2	63.0	89.5

Sources:

V. Johnson (1960), NBS, Wright Air Development Div. (WADD) Technical Report 60-56, Part II. US Government Printing Office, Washington, DC.
D. H. J. Goodall (1970), A.P.T. Division, Culham Science Center, Abingdon, Oxfordshire, UK.

[a] The high thermal conductivity of nearly pure metals is variable and strongly depends on their impurity content; see Sec. 6.4.2.