SRF Cavities I

IAS, July 13, 2023

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Outline of SRF Cavity Lectures

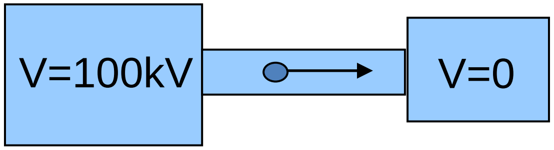
- SRF Cavities - I – July 13, 2023
  - Introduction to EM fields in resonators
  - Pill-box cavities, elliptical cavities, TEM mode cavities
  - Linac architecture and choice of cavity and frequency
  - Other cavity types
  - Cavity circuit representation
- SRF Cavities -II – July 14, 2023
  - Superconducting RF resistance
  - Fundamental parameters of RF resonators (Q-value, shunt impedance, geometry actor, stored energy, transit time factor)
  - Achieving peak performance (extrinsic issues and mitigations)
  - State of the art
- I will cover these slides

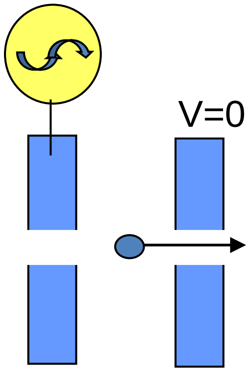
Introduction to EM Fields in Resonators

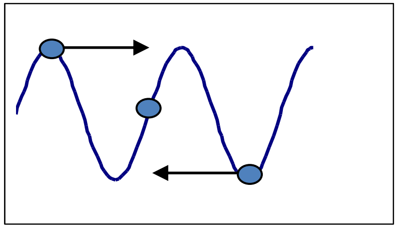
Physics of acceleration

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

- Electric or magnetic fields can act on charged particles – **only E fields can accelerate** – B fields provide a force transverse to the motion
- The accelerating electric fields can be static or time-varying
- Time-varying EM fields mean that acceleration of the charged particle occurs only at specific times (at other times the particles could be decelerated or see no field at all)
- Radio Frequency acceleration demands a bunch structure for the charged particles synchronized with the oscillating field

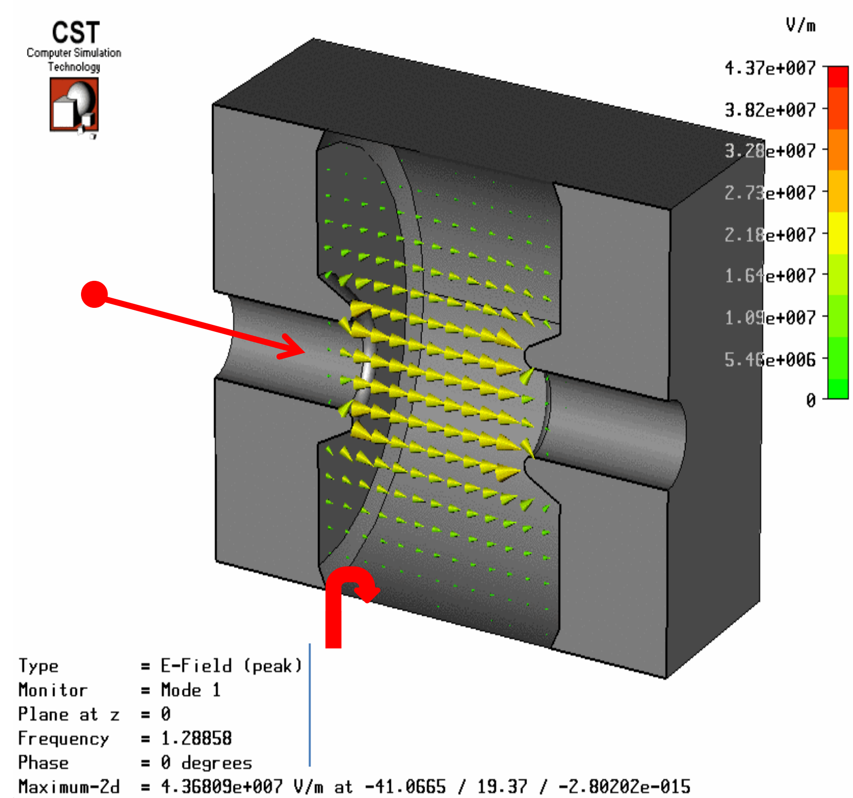






RF Cavities

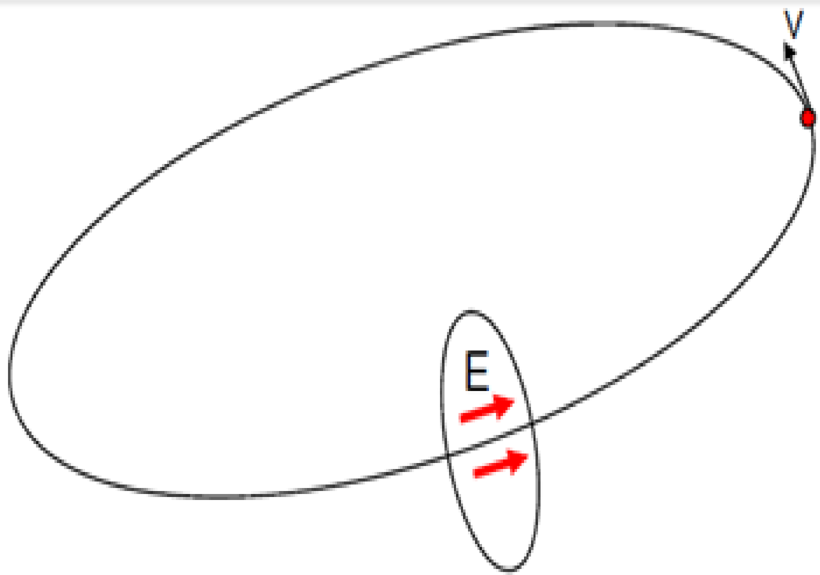
- RF acceleration is typically accomplished in `rf cavities’ or `resonators’; specially designed structures with electrically conductive walls
- The cavity is sized to resonate at a particular rf frequency and with a shape such that an electric field is produced along the path of the charged particle as it passes through the cavity
- A small driving rf signal couples electro-magnetic energy into the cavity to resonantly grow the accelerating field.



$$F = qE(t) = qE_0 \cos(\omega t)$$

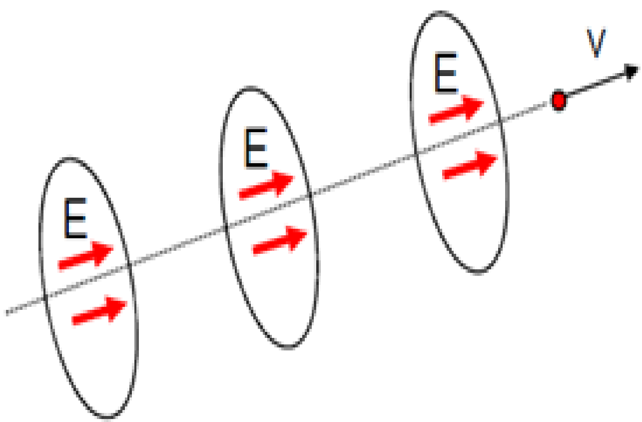
Circulating and linear variants

There are two main approaches for accelerating with time-varying fields



Circulating Accelerators

- Use multiple passes through a small number of cavities (cyclotrons, synchrotrons, microtrons, …)



Linear Accelerators

- Use a single pass through a large number of cavities

Simple in concept – the devil is in the details

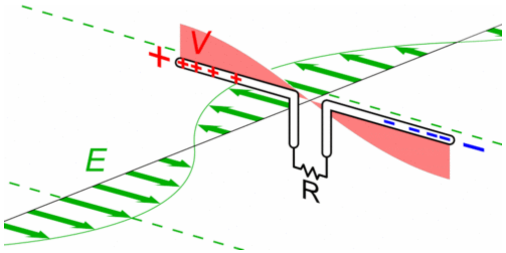
RF Frequency (Wavelength)

Radio frequency (**RF**) is the oscillation rate of an alternating current or voltage or of an electromagnetic field in the frequency range from around 20kHz to around 300 GHz. The useful range for rf particle accelerators is from 3MHz to 30GHz.

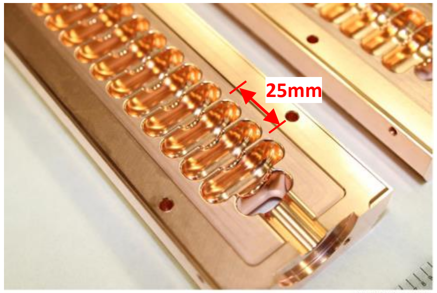
Frequency range	Wavelength range	Designation
3–30 MHz	100–10 m	High frequency (HF)
30–300 MHz	10–1 m	Very high frequency (VHF)
300 MHz – 3 GHz	1 m – 10 cm	Ultra high frequency (UHF)
3GHz – 30GHz	10cm-1cm	Super High Frequency (SHF)

We will mention through the next several lectures why this range is practical for acceleration.

ISAC 35MHz



CLIC 12GHz



SLAC-CERN

Useful RF relations

✓

In free space $$f\lambda = c$$

where f is the rf frequency (Hz or cycles per second), λ (m) the wavelength and c is the speed of light (2.998×10^8 m/sec).

Other useful relations

1 rad/sec ≠ 1 Hz

$$\omega = 2\pi f = \frac{2\pi c}{\lambda} \quad \text{where } \omega \text{ is frequency in radians per sec}$$

$$\omega = kc \quad \text{where} \quad k\ (\mathrm{m}^{-1}) = \frac{2\pi}{\lambda} \quad \text{is the wave number}$$

(rad/m)

Other useful constants are $\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{m^{-3}\,kg^{-1}\,s^4\,A^2} = 8.85 \times 10^{-12}$ F/m
and $\mu_0 = 1.26 \times 10^{-6}\ \mathrm{m\,kg\,s^{-2}\,A^{-2}} = 4\pi \times 10^{-7}$ H/m

and

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad \text{and} \quad Z_0 = \frac{|E|}{|H|} = \eta = \mu_0 c = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega$$

Characteristic impedance of free space

Wave equation and boundary conditions

- Maxwell's equations can be used to derive the wave equations in free space

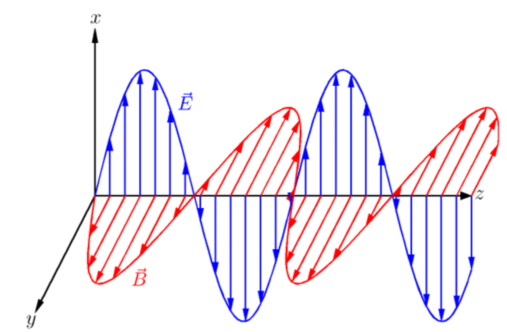
Current density
Charge density

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad \text{and} \quad \nabla^2 \vec{B} - \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} = 0 \quad \text{for} \quad \mu_0 \varepsilon_0 = \frac{1}{c^2} \quad \text{and} \quad \vec{J} = \rho = 0$$

- With solutions (assuming wave is travelling along z axis)

$$\overrightarrow{E_x} = \overrightarrow{E_{x0}} \cos(kz - \omega t + \phi_0) \quad \text{and} \quad \overrightarrow{B_y} = \overrightarrow{B_{y0}} \cos(kz - \omega t + \phi_0)$$

$$\text{with} \quad \omega = 2\pi f, \quad k = \frac{2\pi}{\lambda}, \quad \frac{E_{x0}}{B_{y0}} = c$$



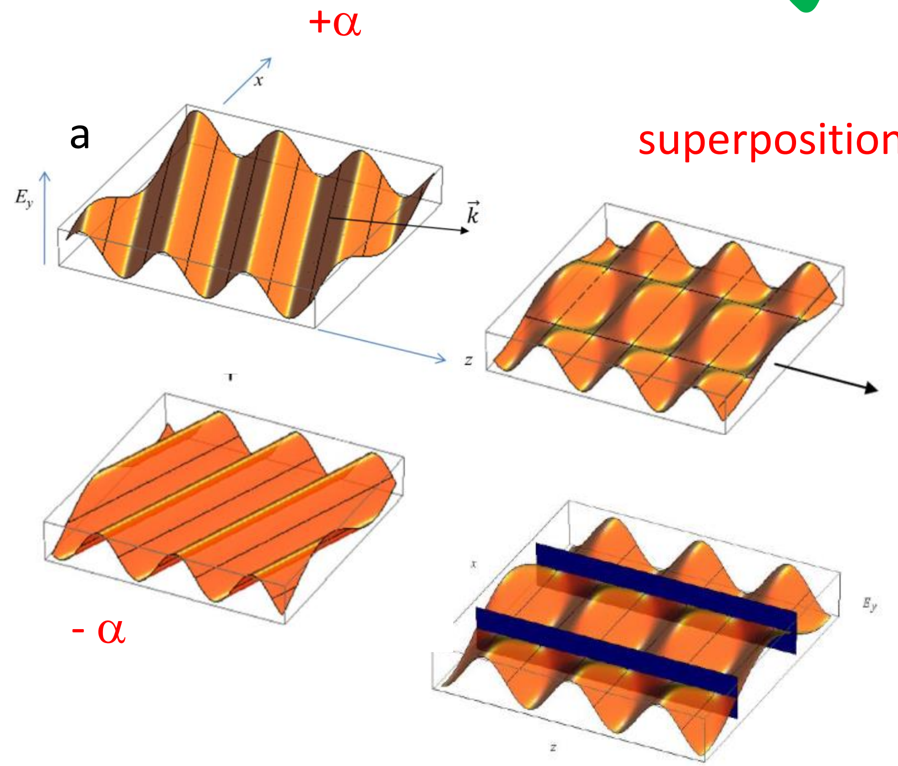
- The waves reflect at a conducting wall governed by the boundary conditions

$$\hat{n} \times \vec{E} = 0 \ \text{and} \ \hat{n} \cdot \vec{H} = 0$$ where $\hat{n}$ is normal to the surface

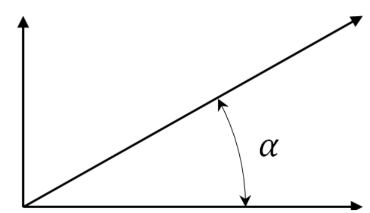
- *Hence **E*** is **perpendicular** to a conductor and ***H*** is **parallel**

Superposition of waves

- Consider two waves propagating at an angle $\pm\alpha$
- the waves will superimpose to produce standing waves with the spacing between the nodes given by the wave frequency and the wave-front angle
- These nodes are a distant $a=\pi/k_\perp$ apart where $k_\perp$ is the x-component of k vector (angular wave number $2\pi/\lambda$)
- Note that two virtual parallel conducting boundaries can be positioned at the nodes such that they do not interfere with the pattern
- Note also that an integer multiple of nodes are possible between conducting planes or $k_\perp a = m\pi$



$k_\perp = k \sin\alpha$ $\qquad k = \frac{\omega}{c}$



$k_z = k \cos\alpha$

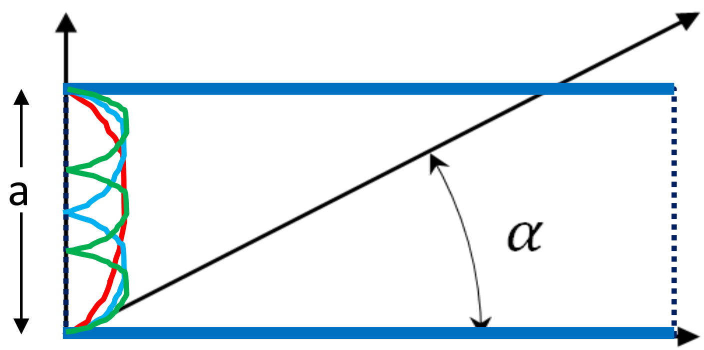
Wave-guide modes (2D)

$$k^2 = k_\perp^2 + k_z^2 = \left(\frac{m\pi}{a}\right)^2 + k_z^2 = \frac{\omega^2}{c^2}$$

At α=90 deg and m=1 then $\omega = \omega_c$ corresponding to the lowest frequency that can propagate – cut-off frequency

$$k_\perp = \frac{\omega_c}{c} = k \sin\alpha \qquad\qquad k = \frac{\omega}{c}$$

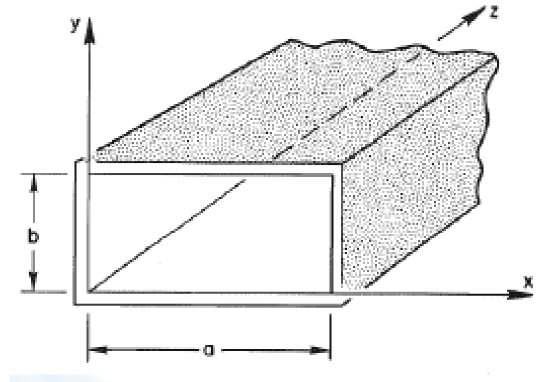
m=1,2,3

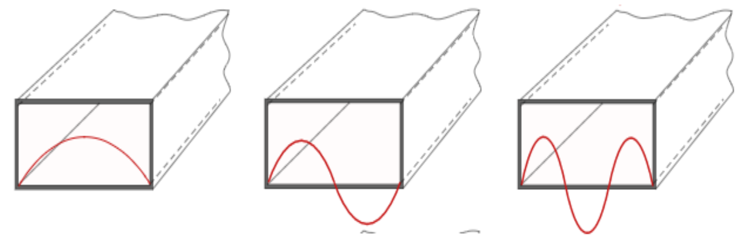


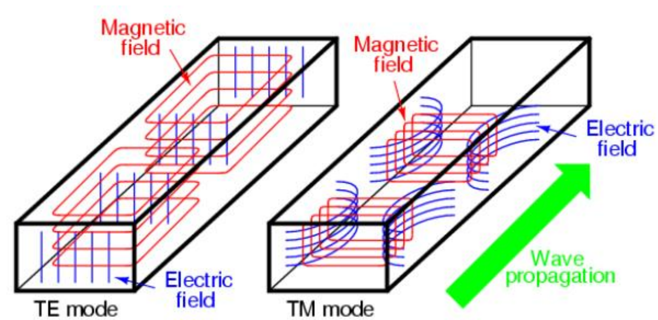
$$k_z = \frac{\omega}{c}\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} = k \cos\alpha$$

Waveguides and modes

- Now consider a rectangular conductor of cross-section *a* x *b* and infinite length – such a structure `waveguide’ can support an infinite series of EM modes that satisfy the transverse boundary conditions
- Two families of solutions exist in rectangular waveguides
  - TE (transverse electric) modes – electric field is always perpendicular to the direction of propagation – Ez (z,t) = 0
  - TM (transverse magnetic) modes – magnetic field is always perpendicular to the direction of propagation - Bz (z,t) = 0

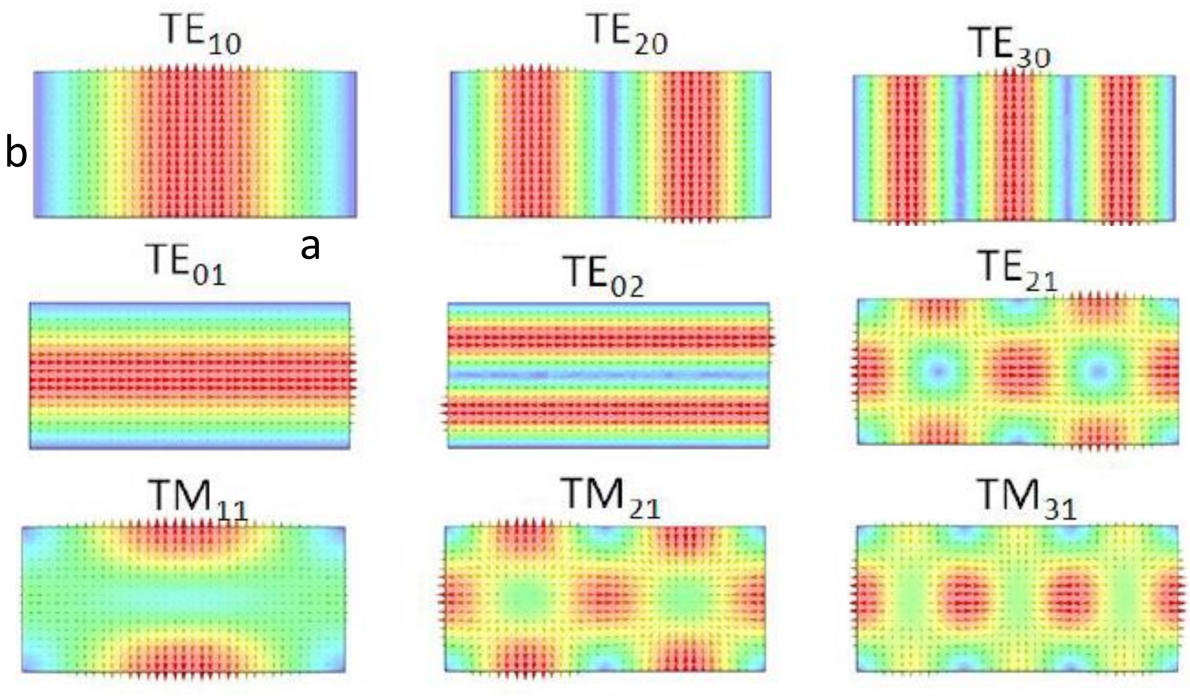


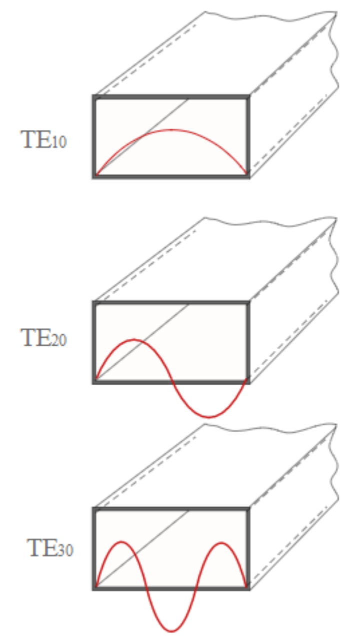




Naming modes and mode frequencies in rectangular WG

The waveguide modes are named after the family (TE or TM) and on the number of half wave patterns along each transverse axis TE_{mn} and TM_{mn}
TE modes have indices m=0, 1, 2, … n=0, 1, 2, … with m=n=0 not allowed and TM modes have indices m=1, 2, 3, … and n=1, 2, 3, …





The mode frequency is given by:

$$k^2 = \left(\frac{\omega}{c}\right)^2 = k_\perp^2 + k_z^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + k_z^2 .$$

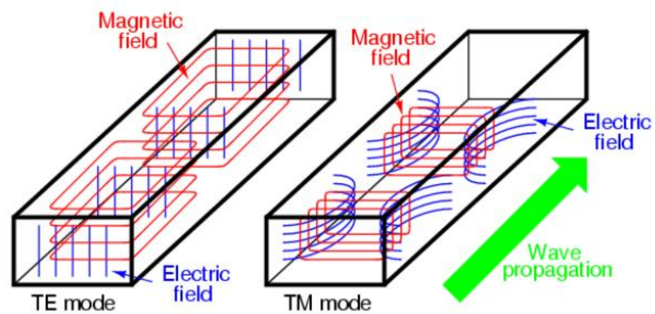
k_z is the wave number in the propagation direction

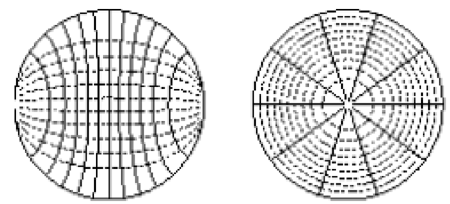
$$k_z = \frac{2\pi}{\lambda_z}$$

Common Waveguides

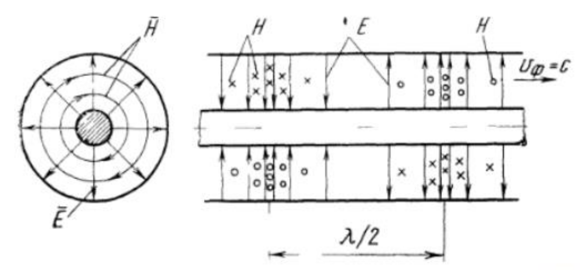
Two types of transmission lines are typically used in accelerator systems: rectangular or circular waveguide and a coaxial line

- Waveguides
  - Typically rectangular or circular
  - Can support TE and TM modes
  - Usually the lowest mode, TE10 [rectangular] or TE11 [circular] are used and the rf range (bandwidth) is limited by the cut-off frequencies of this and the next lowest modes (next slide)
- Coaxial line
  - The coaxial line has two conductors, center and outer, and therefore can support **TEM mode** (as well as waveguide modes).





(1) TE_{11} (2) TM_{01}



Waveguide cut-off frequency

The cut-off frequency of an electromagnetic waveguide is the lowest frequency (longest wavelength) for which a mode will propagate. Below the cut-off frequency, the longitudinal wave number is imaginary. In this case, the field decays exponentially along the waveguide – evanescent wave

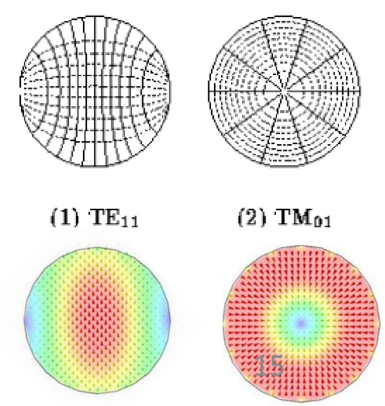
- For a rectangular waveguide, the cut-off frequency is where $k_z = 0$

$$\omega_c = c\sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2} \qquad \text{For } \mathrm{TE}_{10} \quad f_c = \frac{c}{2a} \quad \text{and} \quad \lambda_c = 2a$$

where the integers *n, m ≥ 0* are the mode numbers and *a* and *b* are the lengths of the sides of the waveguide

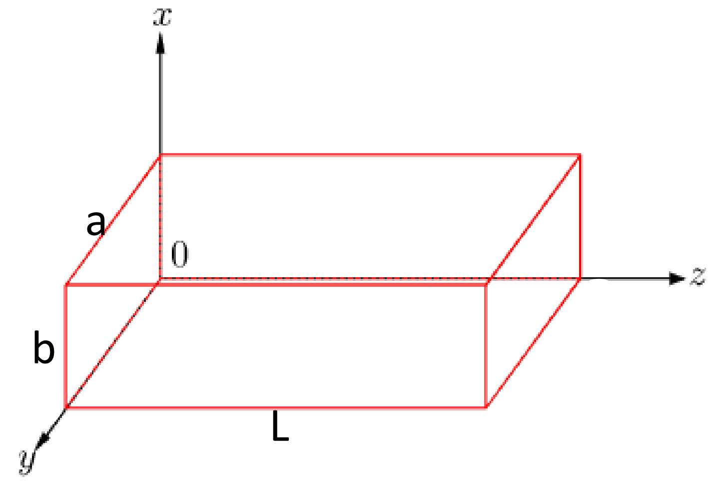
- For a cylindrical waveguide the indices m and n (for TM_{mn} and TE_{mn}) correspond to number of azimuthal and radial nodes respectively
- the cut-off frequencies are defined by the nodes of the relevant Bessel functions. The lowest modes are:

$$TE_{11} \;\rightarrow\; \omega_c = 1.84\frac{c}{r} \qquad TM_{01} \;\rightarrow\; \omega_c = 2.405\frac{c}{r}$$

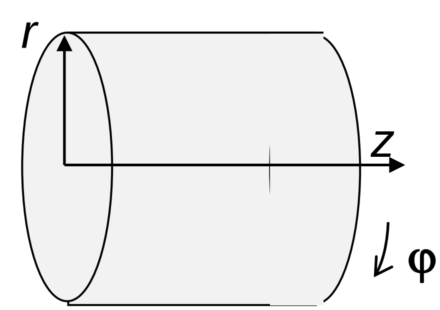


Cavity modes

- Consider a rectangular conducting box of dimension *a* x *b* x *L*
- In this case k_z can only take discrete values in order that the boundary condition at z=0 and z=*L* are met
- The modes are defined by three indices given by the number of half wave amplitude variations along each dimension m, n, p
- There are only discrete frequencies that are resonant defined by the cavity dimensions
- A similar analysis can be done for a cylindrical pipe of fixed length
- The cylindrical modes are classified by the nomenclature TE_{mnp} or TM_{mnp}. The integers *m,n,p* are measures of the number of nodes E_z or B_z undergoes in the φ, *r* and *z* directions, respectively.



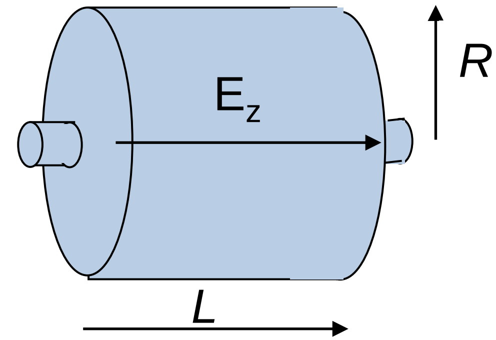
$$\omega = c\sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2 + \left(\frac{p\pi}{L}\right)^2}$$

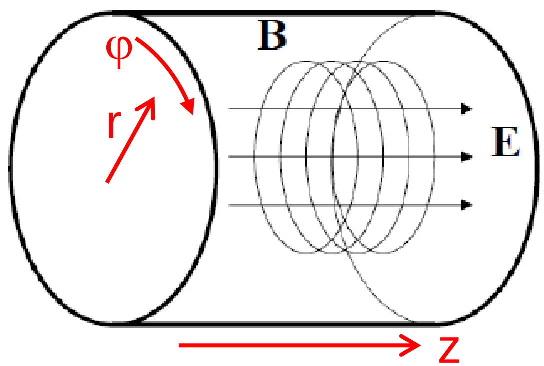


Pill-box cavities, Elliptical cavities, TEM mode Cavities

Pill Box – accelerating TM_{010} mode

- In order to be accelerated a charge particle has to interact with an electric field in the direction of travel
- The cylindrical mode typically used for acceleration in a pill box is TM_{010}



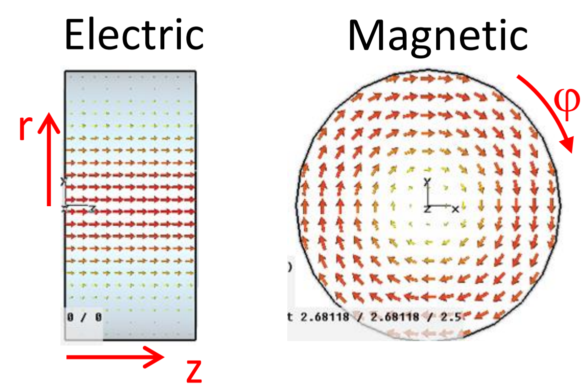


$$E_z = E_0 J_0\left(\frac{2.405r}{R}\right)e^{-j\omega t}$$

$$H_\varphi = \frac{-j}{\eta} E_0 J_1\left(\frac{2.405r}{R}\right)e^{-j\omega t}$$

$$E_\varphi = E_r = H_z = H_r = 0$$

$$\omega_{010} = \frac{2.405c}{R} \qquad \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$



where J_0 and J_1 are Bessel functions of the first kind

$$J_\alpha(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma(m+\alpha+1)}\left(\frac{x}{2}\right)^{2m+\alpha}, \quad \Gamma(n) = (n-1)!$$

Pill Box – Other modes – TM_{0n0}

- The TM_{0n0} series are the accelerating modes (non-zero E field on axis) with no variations of fields in the z direction (frequency does not depend on length
- Frequency is dependent on radius – higher modes have higher frequencies
- The non-zero field components are

$$E_{zn}(r,t) = E_{0n} J_0(k_n r)\cos(\omega_n t)$$

$$H_{\theta n}(r,t) = -j\frac{E_{0n}}{\eta} J_1(k_n r)\sin(\omega_n t)$$

- Mode frequencies given by

$$\omega_{TMmnp} = c\sqrt{\left(\frac{x_{mn}}{R}\right)^2 + \left(\frac{p\pi}{L}\right)^2}$$

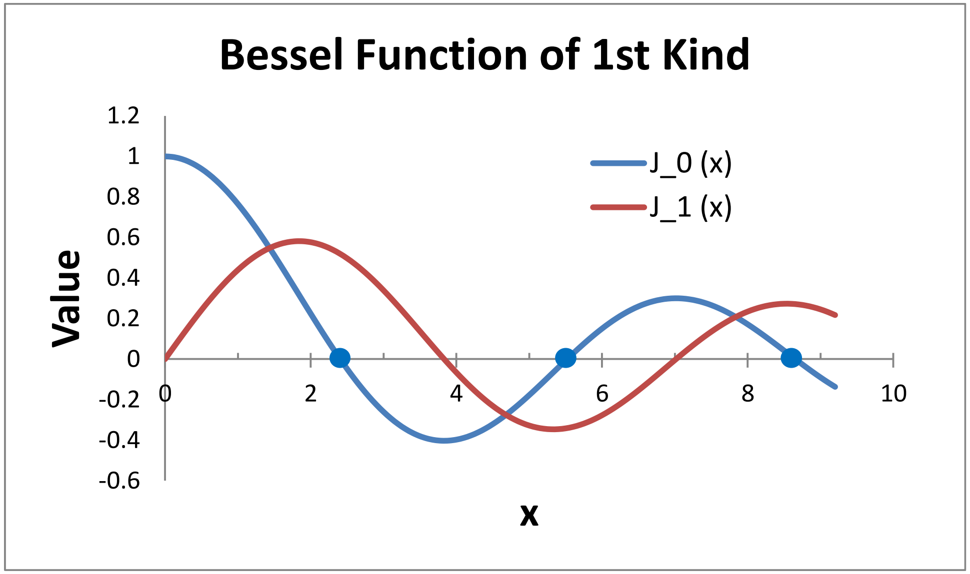
x_{mn} is the n$^{\text{th}}$ root of J_m

Mode	k_n
TM_{010}	2.405/R
TM_{020}	5.520/R
TM_{030}	8.654/R

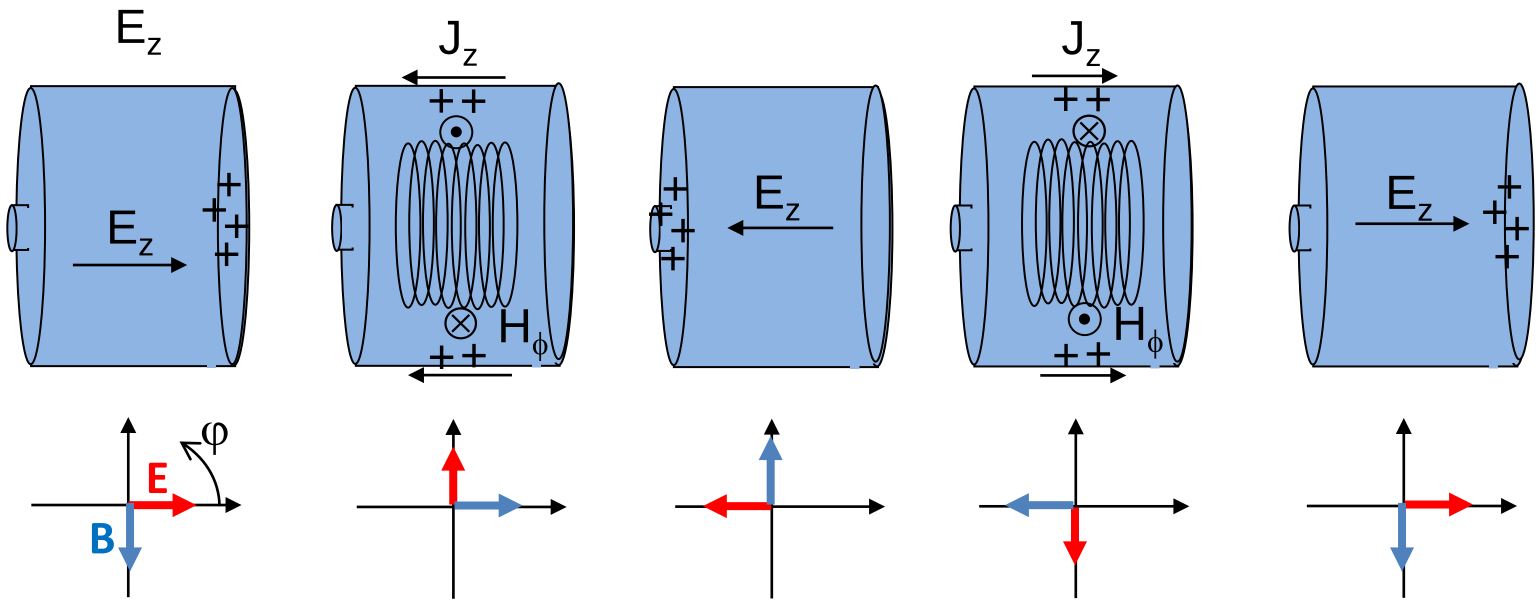
$$\omega_n = ck_n \text{ or } k_n = \frac{2\pi}{\lambda_n}$$

where k_n is valid only for zeros of J_0

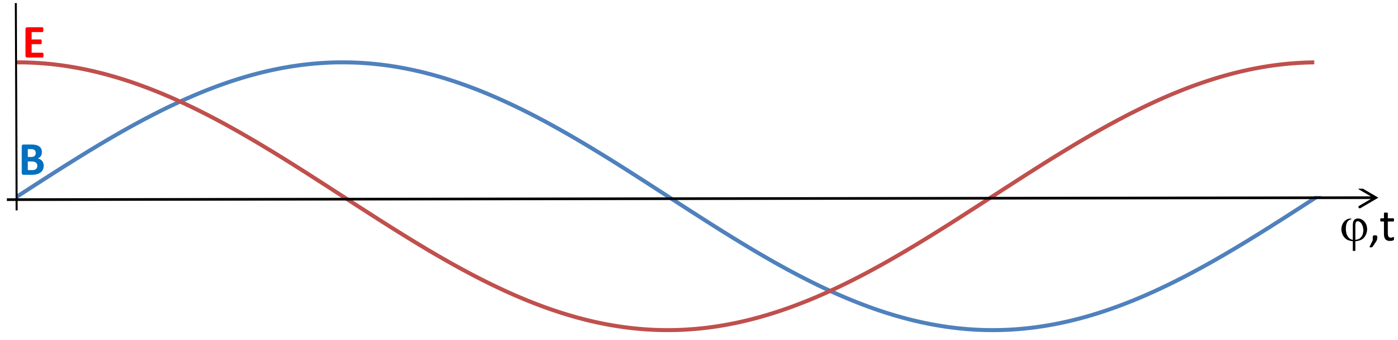
Example: TM_{010} - For R=0.1m, k1=24.05m^{-1}=2π/λ
So λ=0.26m and freq = 3e8/0.26 = 1.1GHz
Also for TM_{020} freq = 2.64 GHz, λ=.11m



Time Variation of TM_{010} Fields in pill-box



Electric field and magnetic field are time varying and 90 degrees out of phase. The field `phase' can be plotted as a rotating vector in complex space with the field amplitude given by the projection on the horizontal axis.



$$\varphi = \omega t \quad \text{(rf phase)}$$

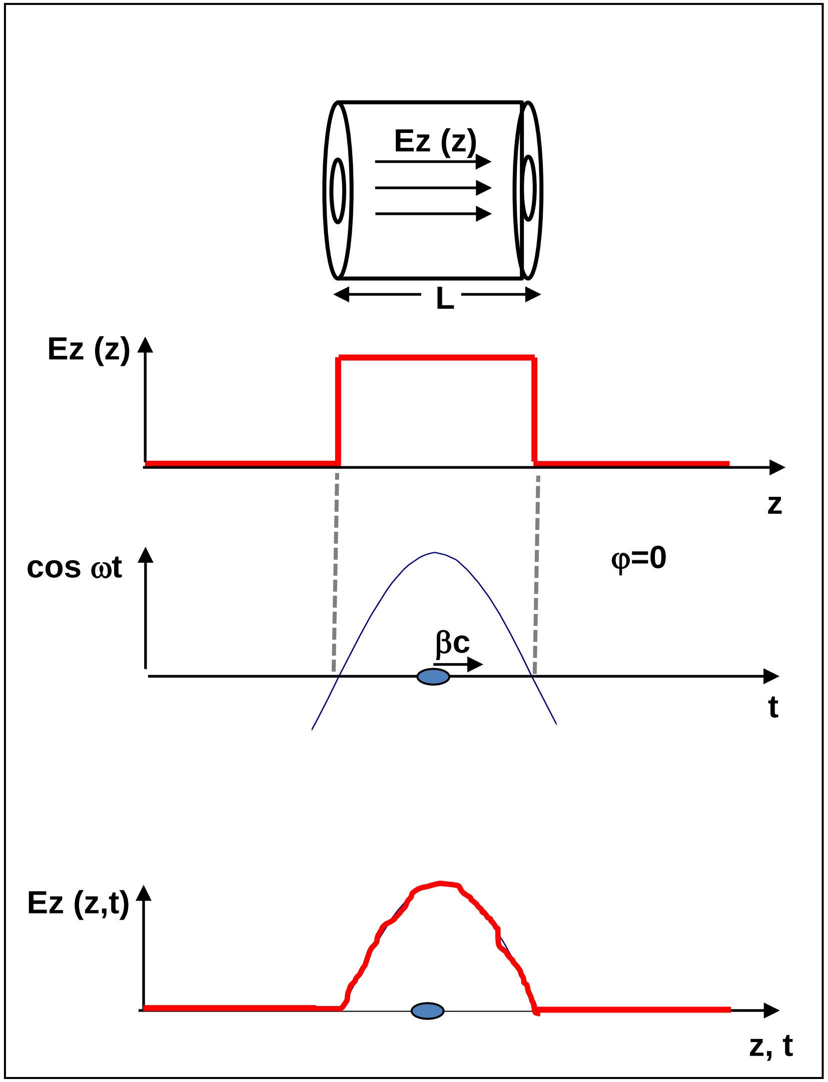
Acceleration in a pill-box

- A pill-box cavity in TM010 mode has the electrical field on axis that is (spatially) constant along the cavity length
- A hole (beam-pipe) can be placed at the entrance and exit to allow charged particles to pass through
- The field amplitude varies with time (radio-frequency) and so the pill-box length is constrained by the oscillation frequency and the speed of the particle
- The length that the particle goes in half an rf cycle is given by

$$v\Delta t = \beta cT/2 = \beta c/(2f) = \beta\lambda/2$$

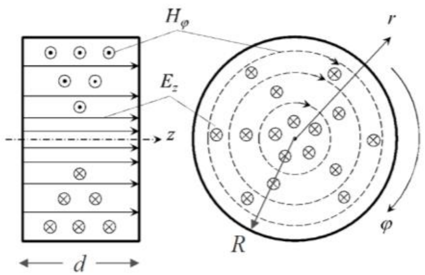
where T is the rf period and v is the velocity

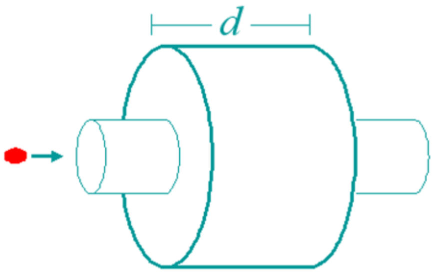
- So for maximum acceleration the pill-box can only be $L_{max}=\beta\lambda/2$

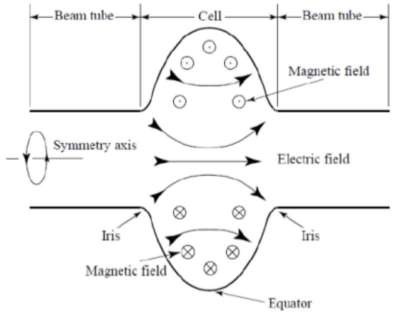


Pill-box -> Elliptical

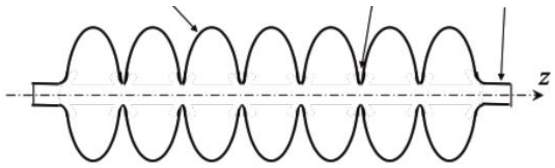
- A popular cavity style is the elliptical cavity that uses the TM010 mode

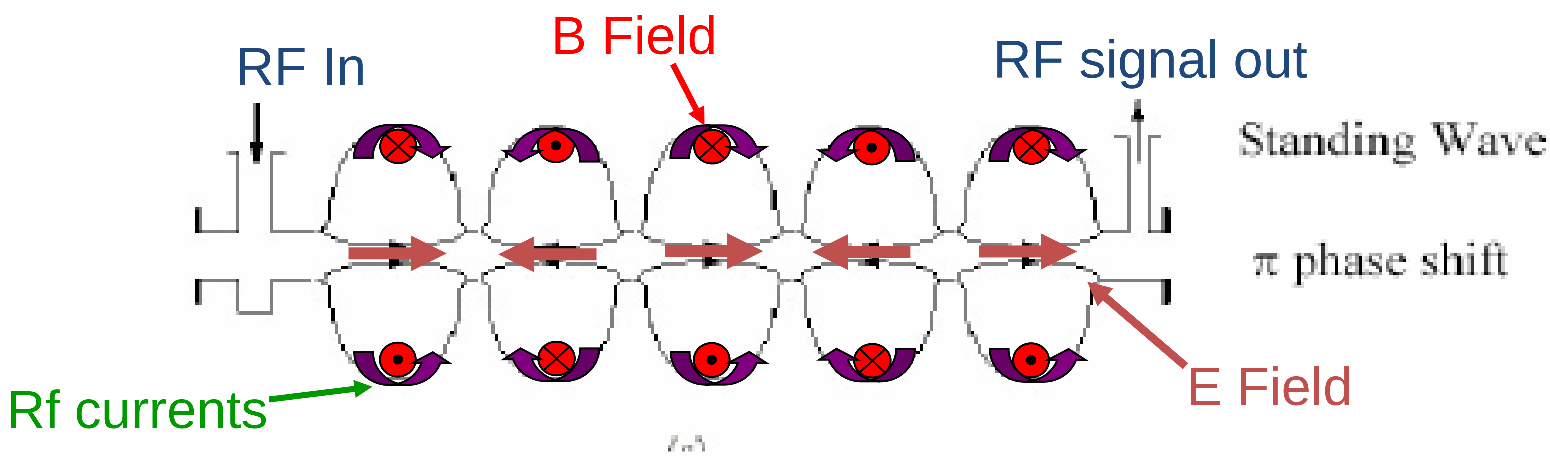






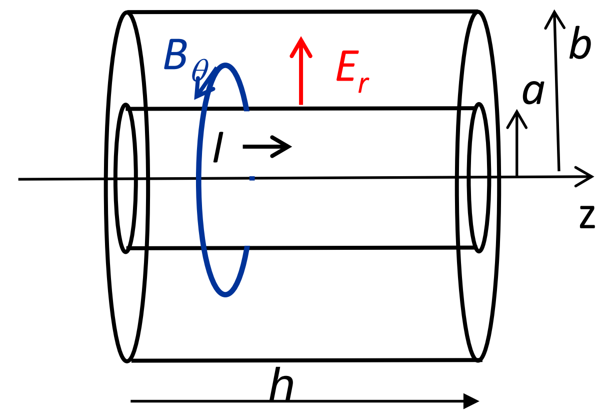
- The cavity is typically made as a multi-cell cavity operating in \`pi' mode (180 degrees from cell to cell and each cell center separated by $\beta\lambda/2$





Coaxial resonator (TEM mode)

- Coaxial geometries support TEM modes - assume inner radius *a* and outer radius *b* and height *h* with grounded plates at the ends
- A standing wave occurs with E_r vanishing on the end walls at *z=0* and *z=h* with non-zero field components



$$B_\theta = \frac{\mu_0 I_0}{\pi r} \cos \frac{p\pi z}{h} e^{j\omega t}$$

$$E_r = -j \frac{\eta I_0}{\pi r} \sin \frac{p\pi z}{h} e^{j\omega t}$$

$$\text{where } \omega = k_z c = \frac{p\pi c}{h}, \quad p = 1, 2, 3, \ldots$$

$$\text{and} \quad \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

The voltage on the inner conductor is given by:

$$V_0(z) = \int_a^b E_r(z) dr$$

$$V_0(z) = \eta \frac{I_0}{\pi} \ln\left(\frac{b}{a}\right) \sin \frac{p\pi z}{h}$$

Example: Half Wave Resonator

- The lowest mode corresponds to p=1 (half-wave resonator)

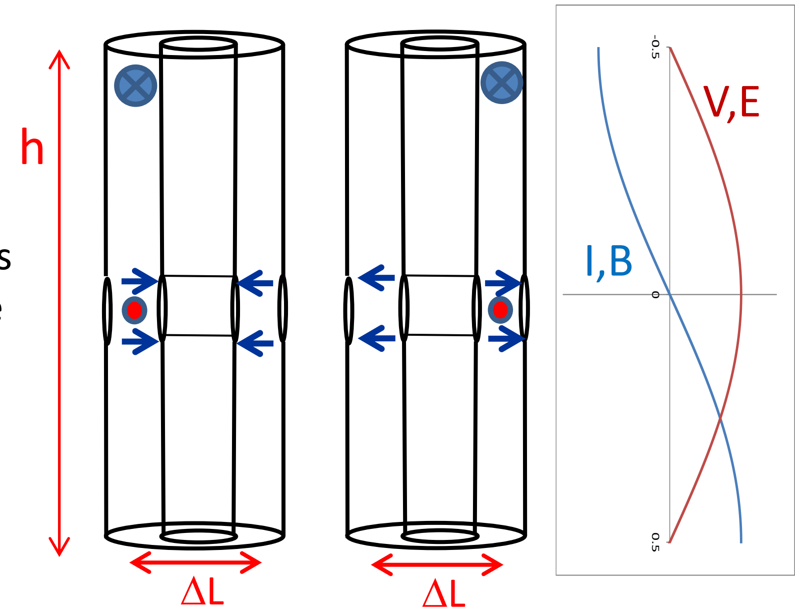
$$f = \frac{pc}{2h}, \quad \lambda = \frac{c}{f} = \frac{2h}{p} \quad \text{for} \quad p = 1, 2, 3, \ldots$$

- An accelerating cavity can be made by forming beam ports in the outer and inner conductors at the center where the E_r field and voltage is maximum
- The ion arrival time is arranged so that the E_r field is maximum when the ion crosses the first gap and undergoes a π phase shift as the ion travels to the second gap
- Note for synchronism there is a relation between gap to gap distance, particle velocity and rf frequency

$$\Delta t = \frac{\Delta L}{v} = \frac{\Delta L}{\beta c}$$

so for synchronism with rf period T

$$\Delta t = \frac{T}{2} = \frac{1}{2f} = \frac{\lambda}{2c} \quad \text{so} \quad \Delta L = \beta c \frac{\lambda}{2c} = \frac{\beta\lambda}{2}$$



Example: h = 0.5 m, λ=1m, f=300MHz

If β=0.2 then the gap to gap distance must be ΔL=0.1m for synchronism

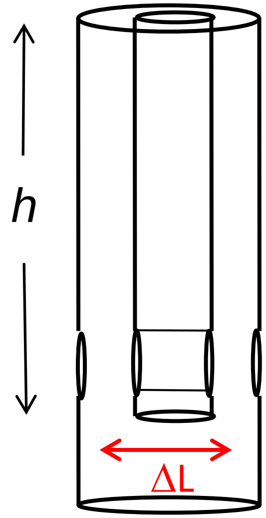
In other words, a particle traveling at v=0.2c in a 300MHz field sees a field reversal every 10 cm

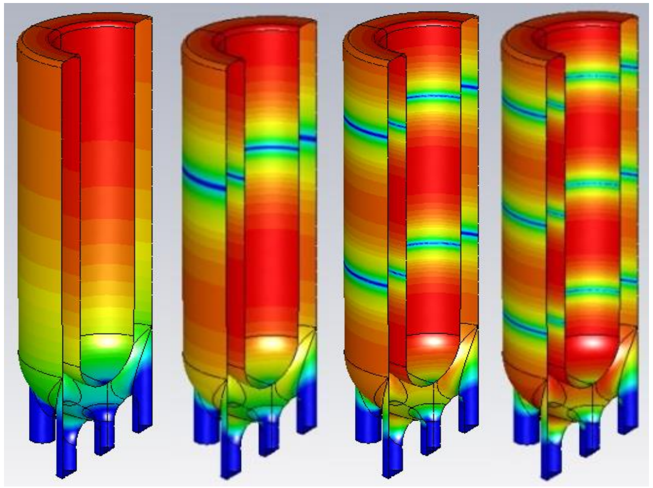
Example: Coaxial quarter wave resonator (QWR) ✓

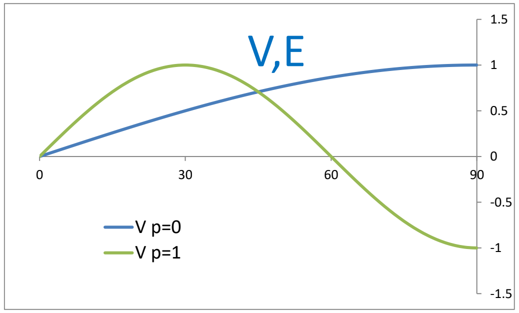
- Another popular coaxial TEM mode cavity is the quarter wave resonator
- Here the inner conductor is open at one end with a resonant length of *h=λ(1+2p)/4* where *p*=0,1,2

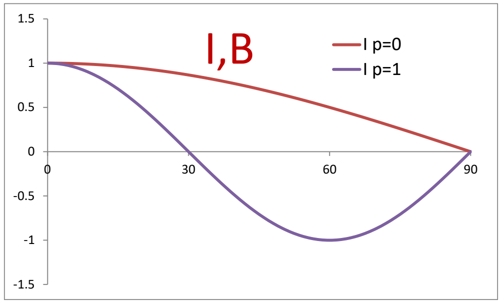
$$f = \frac{1+2p}{4h}c \quad \text{for} \quad p = 0,1,2$$

- The most popular accelerating mode has p=0
- The maximum voltage builds up on the open tip and the maximum current is at the root
- A beam tube is arranged near the end of the tip to produce a high voltage double gap acceleration geometry similar to the half wave resonator







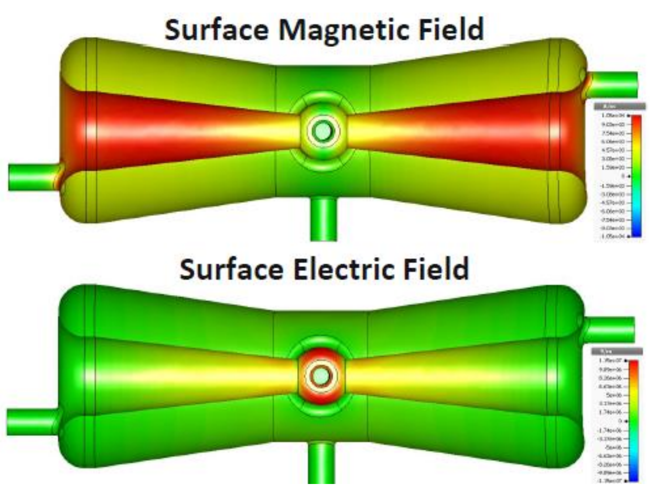


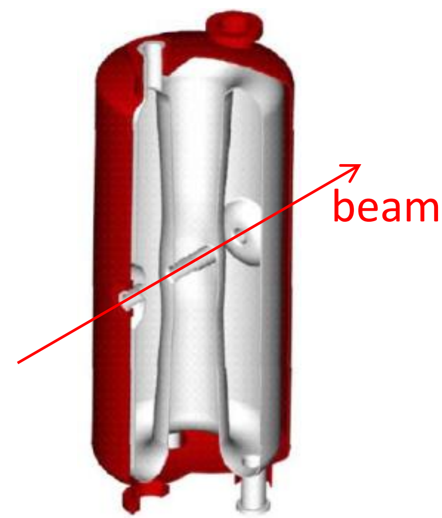
Example: h = 0.5 m, λ=2m, f_0=150MHz

If β=0.1 then the gap to gap distance must be ΔL=0.1m for synchronism

RF Structures – Field Computation

- In all but trivial cases analytic solutions for rf fields in conducting structure are not available and cavities are designed to optimize performance
- For example, surfaces are shaped to minimize peak electric fields and peak magnetic fields on the surface while maximizing acceleration
- Computer codes (CST, COMSOL, HFSS, …) are used to calculate the resonant frequency and field strength (electric and magnetic) of the modes of interest based on Maxwell's Equations and the boundary conditions – *E* is perpendicular to a conductor and *H* is parallel





Linac architecture and choice of cavity and frequency

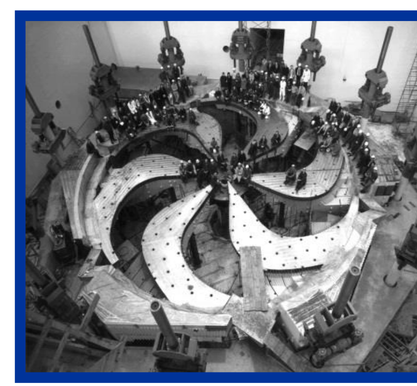
Accelerating Electrons vs Hadrons

Electron and hadron linacs are distinctly different due to the mass of the particles

- Electron – 0.511MeV/c^2
  - 300kV - γ=1.58, β=0.78
  - 550MeV - γ=1011, β=1

ARIEL 300kV e-gun

- Protons – 938 MeV/c^2
  - 300kV - γ=1.003, β=0.025
  - 550MeV - γ=1.58, β=0.78

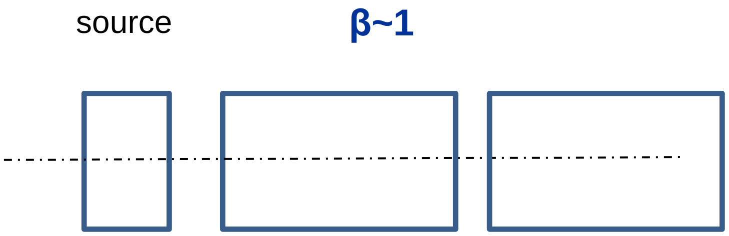


TRIUMF 500MeV cyclotron

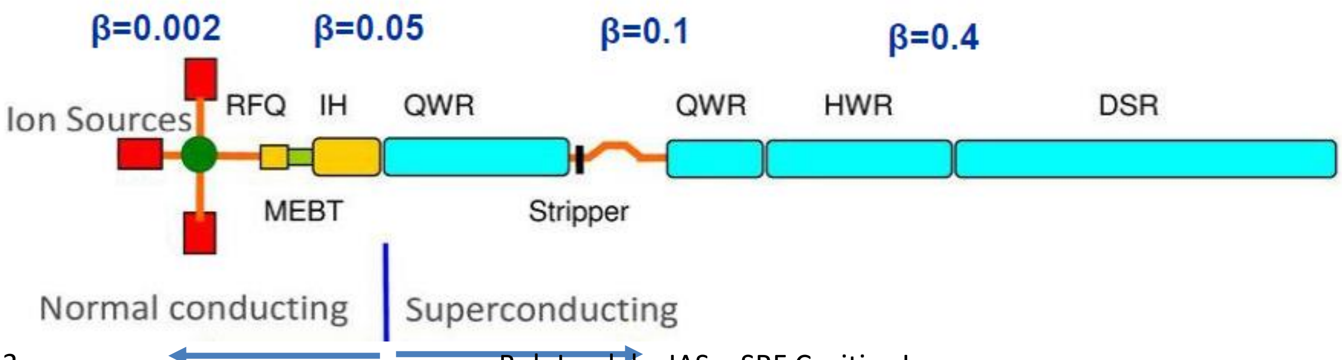
Linac architecture

- RF acceleration depends on synchronization between the rf and the particle bunches so is dependent on velocity
- Electrons become relativistic with only modest voltage (E>1MV -> v/c=0.94) -> electron linacs are designed assuming $v_e=c$
- Design choices include pulsed vs cw, rf frequency, superconducting vs normal conducting, rf architecture

- Electron linacs – common building blocks all designed for $v_e=c$ (β=1)

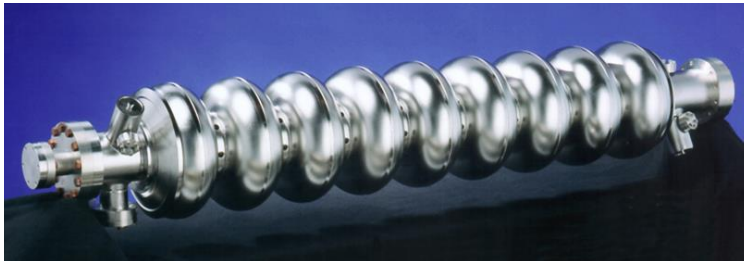


- Hadron linacs – various building blocks – different technologies each optimized to accelerate a given velocity range



Accelerating Electrons vs Hadrons

- SRF electron linac cavities



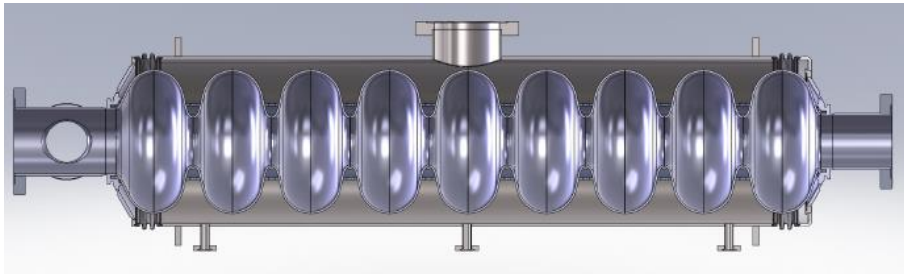
- SRF Hadron linac cavities

SRF Electron acceleration

- The 1.3GHz nine cell elliptical cavity is the dominant cavity for SRF electron linacs (XFEL, LCLS-II, Shine, ARIEL, ELBE, …)
- Used for both pulsed and continuous wave (CW) applications
- Typical gradients for cw application are 16 MV/m (LCLS-II) and 23.5 MV/m for pulsed (XFEL)
- Since beta=1 for electrons with E>1MeV the cavity can be used for all energies from MeV class to GeV class linacs

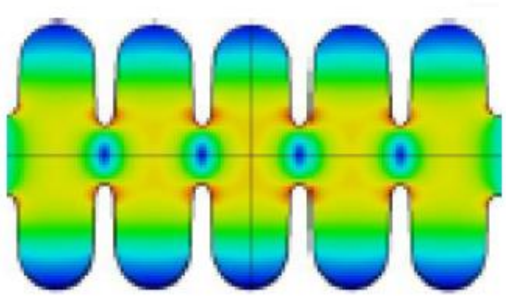
Parameter	Value
RF frequency (GHz)	1.3
Length (m)	1.04
# cells	9
Cell aperture (mm)	70
R/Q (Ohms)	1036
G (Ohms)	270
Ep/Ea	2
Bp/Ea (mT/MV/m)	4.3

More tomorrow

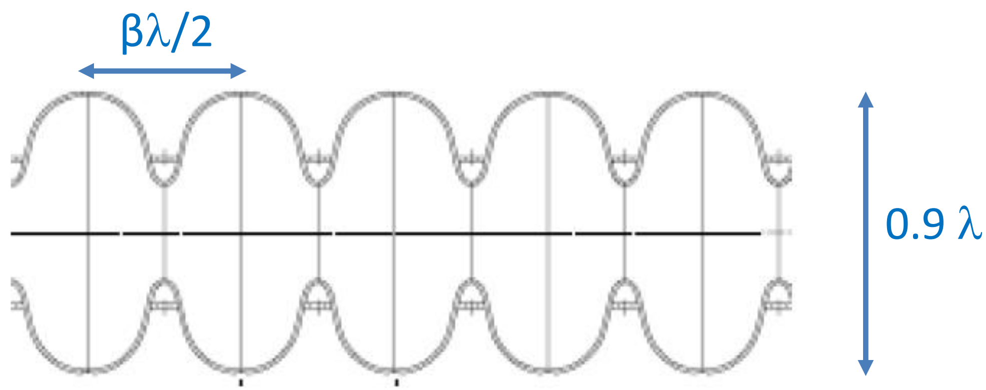


Range of elliptical cavities for beta<1 hadrons

- Elliptical cavities can also be used for energetic hadrons
- Elliptical cavities - in π mode the cell-to-cell distance is $\beta\lambda/2$ but the diameter is ~0.9 λ so cell length/diameter~ $\beta/2$
- At lower velocities the cavity starts to look like a bellows - mechanical stability, multipacting and low rf efficiency are all issues
- Typically, elliptical cavities have $\beta>0.6$
  - ESS (0.67, 0.86), SNS(0.61, 0.81) , PIP-II (0.61, 0.92), FRIB (0.65))

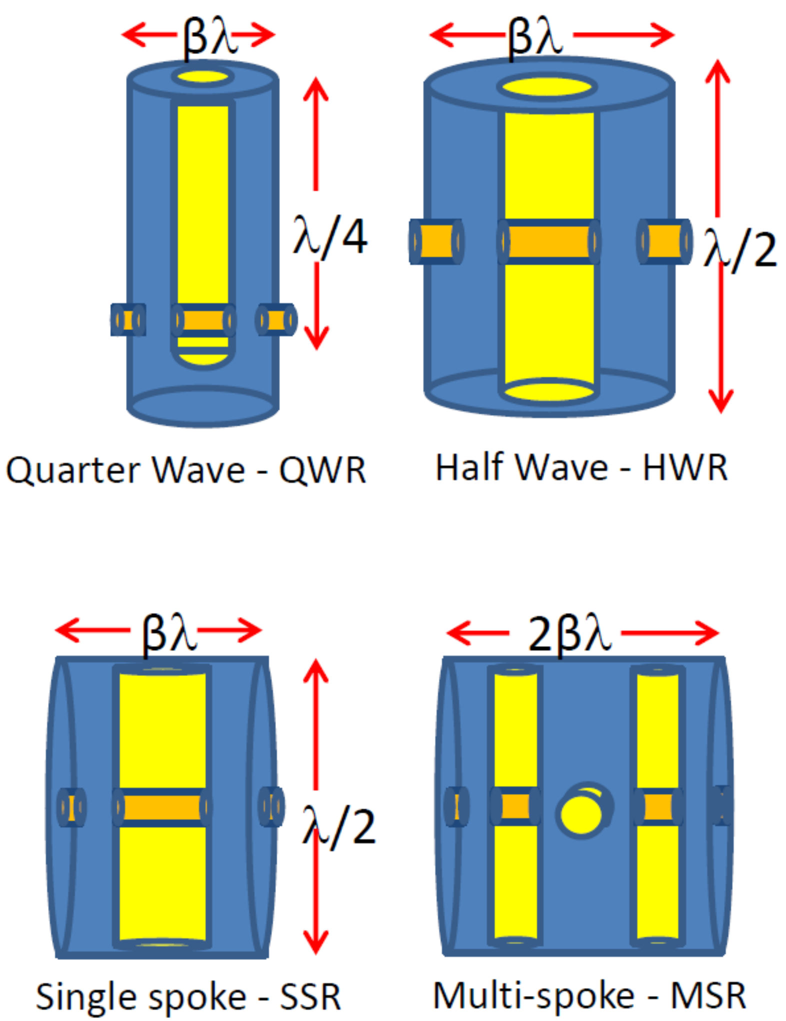


$\beta = 0.61$



$\beta = 1.0$

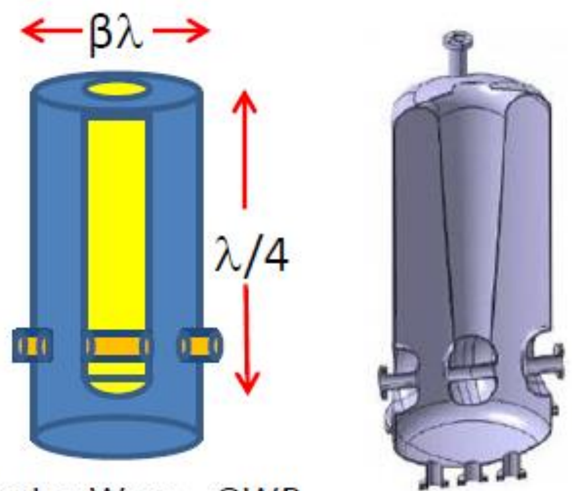
TEM mode cavities (low beta)



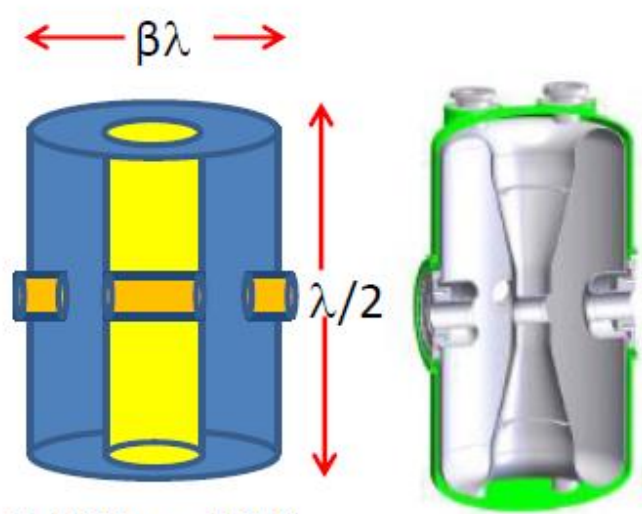
- TEM cavities allow us to accelerate low and medium beta particles in a multi-gap structure with good rf efficiency
- For optimal acceleration gap to gap distances are $\beta\lambda/2$ (synchronism with rf)
- Cavities are defined by the ideal velocity, βo, given by the gap-to-gap distance and the rf frequency
- The cavity height is used to adjust the frequency and the transverse dimensions are scaled to match the velocity
- When β is small then λ must be large for reasonable gap dimensions

QWR vs HWR

- QWR is the cavity of choice for low beta applications (β<0.15)
  - Pros: Requires ~50% less structure and hence less power loss compared to HWR for the same frequency and β_0 – also reduces CM size due to compactness
  - Cons: Asymmetric field pattern produces vertical steering that increases with velocity - Less mechanically stable than HWR due to unsupported end
- HWR is chosen in the mid velocity range (β>0.15) or where steering must be eliminated (ie high intensity light ion applications)
  - Pros: symmetric field pattern and increased mechanical rigidity
  - Cons: 2x rf losses for the same β_0 and λ plus larger length dimension means larger cryomodule



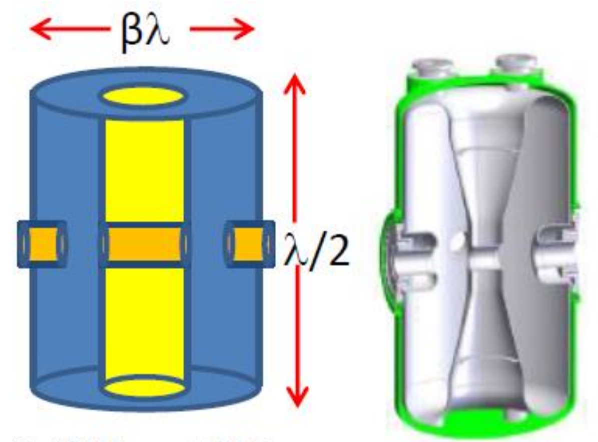
Quarter Wave - QWR



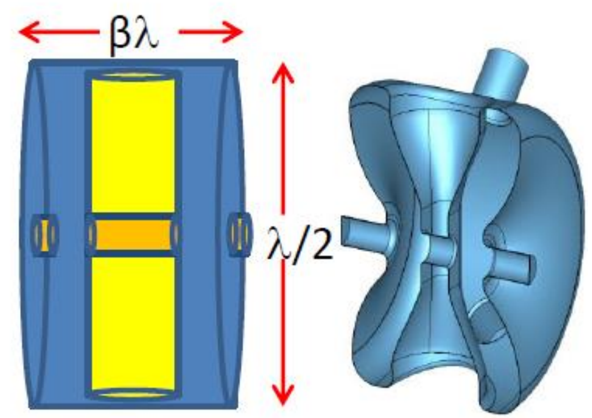
Half Wave - HWR

HWR vs SSR (Single spoke resonator)

- A single spoke resonator (SSR) is another variant of the half-wave TEM mode cavity class
- In HWR the outer conductor is coaxial with the inner conductor (with diameter ~$\beta_0\lambda$) while in the spoke cavities the outer cylinder is co-axial with the beam axis with diameter $\lambda/2$.
- For $\beta_0<0.5$ the SSR has a larger overall physical envelop than the HWR for the same frequency
- For $\beta_0=0.1\text{->}0.25$ HWRs are typically chosen ~160MHz while SSRs are built at ~320MHz.
- For beta $\beta_0=0.25\text{->}0.5$ HWRs and SSRs choose ~320MHz.
- The spoke geometry allows an extension along the beam path to provide multiple spokes in a single resonator giving higher effective voltage but with a narrower transit time acceptance (see SRF Cavity II)



Half Wave - HWR



Single spoke - SSR

ANL HWR and TRIUMF SSR

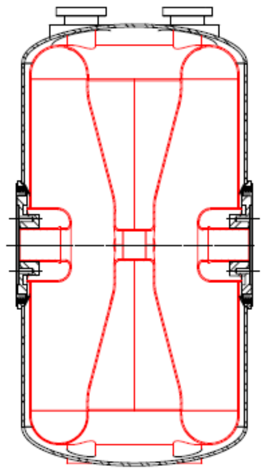
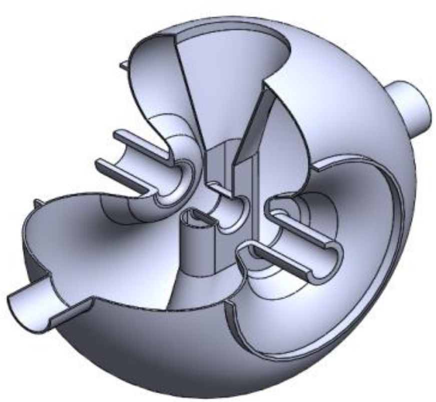
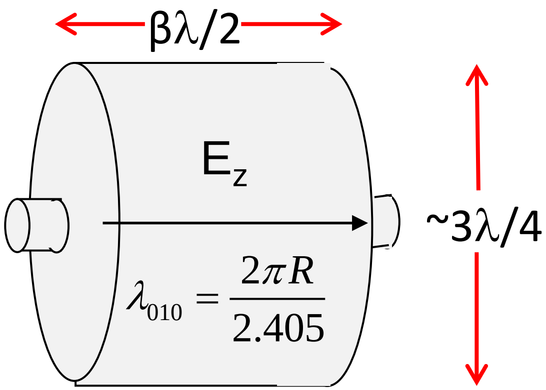


Figure 1: 322 MHz β=0.29 Half Wave Resonator.

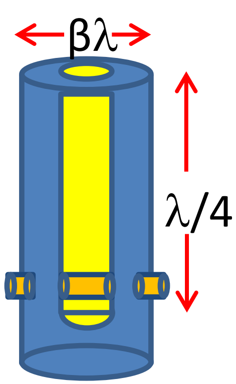


Parameter	HWR	SSR	Units
Frequency	322	325	MHz
β=v/c	0.29	0.3	
$L_{eff} = \beta\lambda$	27	27.7	cm
E_p/E_a	4.3	3.9	
B_p/E_a	7.7	6.3	mT/(MV/m)
G	78	95	Ohms
R_{sh}/Q	224	246	Ohms
Flange to flange	324	390	mm

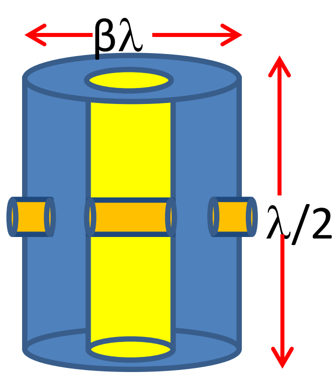
RF Frequency considerations



Pill-box

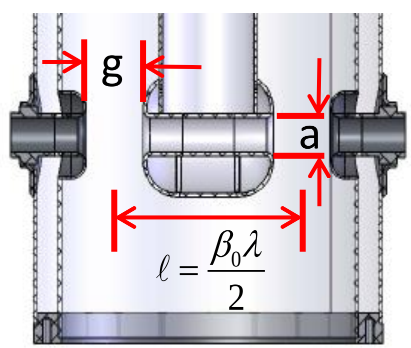


Quarter Wave - QWR



Half Wave - HWR

- Longitudinal (beam direction) cavity dimensions are dependent on beam velocity ($\propto\beta\lambda$) – transverse cavity dimensions are dependent on λ
- Smaller structures are cheaper to fabricate so favour small λ (high frequency)
- Lower frequencies increase size of stable region in longitudinal phase space and increase cavity active length so increase the effective voltage
- Lower velocities need lower frequencies
  - Typically $\beta_0\lambda/2$ between gaps
  - Gap is typically half a cell $\beta_0\lambda/4$
  - Drift tube aperture should be less than the gap to improve acceleration efficiency



g=gap, a=aperture

$$g \approx \frac{\ell}{2} = \frac{\beta_0\lambda}{4} \quad \text{so} \quad a < \frac{\beta_0\lambda}{4} \qquad \lambda > \frac{4a}{\beta_0}$$

ie: for a=0.03m

$\beta_0 = 0.04$ λ>3m f<100MHz

$\beta_0 = 0.12$ λ>1m f<300MHz

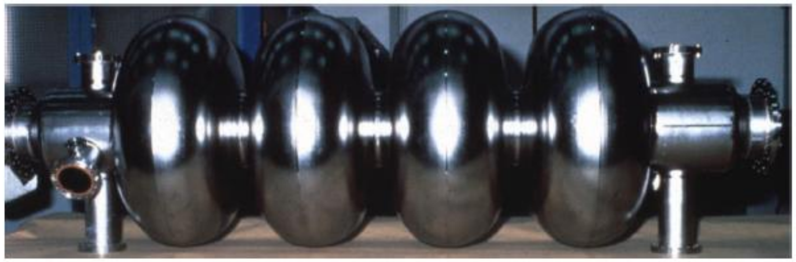
Cavity frequency (cont'd)

- The typical building blocks are QWR, HWR, SSR and MSR but the community has not coalesced towards common designs as has happened in the high beta community
- The main reason is the large parameter space since each project may have different requirements –final energy, ion, existing infrastructure
- At least design techniques, principles, ancillaries are converging
- However at least two frequency series have emerged as being more common for larger projects

fo	h	freq	fo	h	freq
88	1	88	81.25	1	81.25
	2	176		2	162.5
	4	352		4	325
	8	704		8	650
	12	1056		12	975
				16	1300

Cavity examples – elliptical resonators

LEP: 352 MHz



SNS: 805 MHz, $\beta = 0.61$ and 0.81

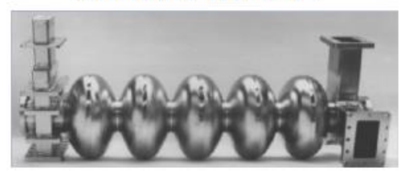
CESR: 500 MHz

KEKB: 508 MHz

TESLA: 1.3 GHz

TRISTAN: 508 MHz

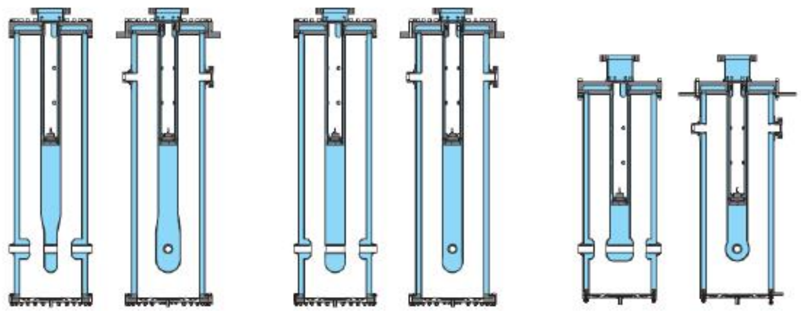
CEBAF: 1.5 GHz



Fermilab: 3.9 GHz

Cavity examples – Quarter wave resonators

TRIUMF ISAC-II Resonators

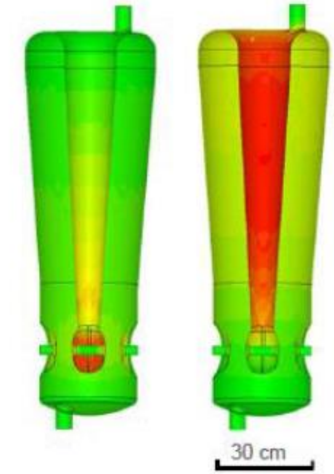


SCB low β (5.7%) 106.08 MHz

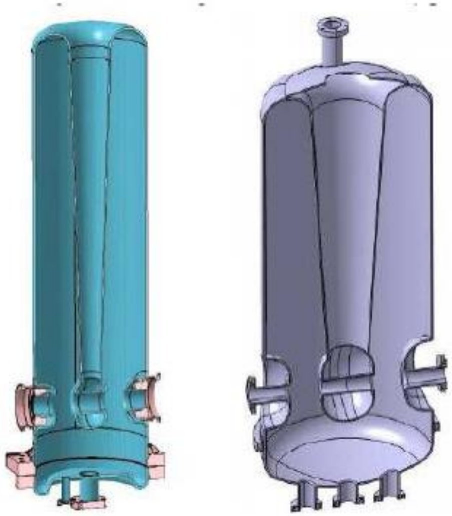
SCB medium β (7.1%) 106.08 MHz

SCC high β (11%) 141.44 MHz

FRIB β=0.041, 0.085 80.5MHz



ANL β=0.077, 0.085 72.5MHz



Spiral-2 β=0.007, 0.12 88.05MHz

RAON β=0.047, 81.25 MHz

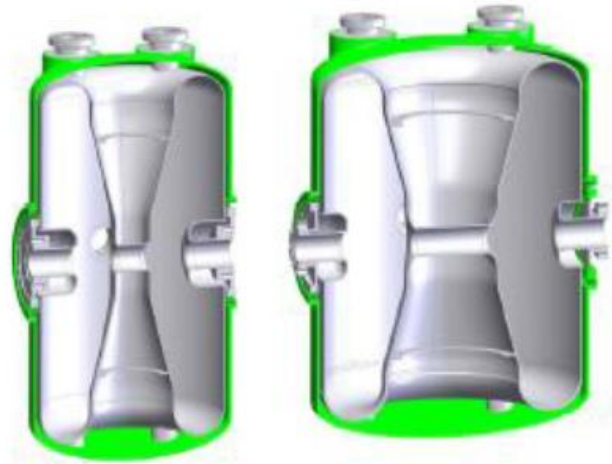
Typical range

β~0.04->0.15

f=50MHz->160MHz

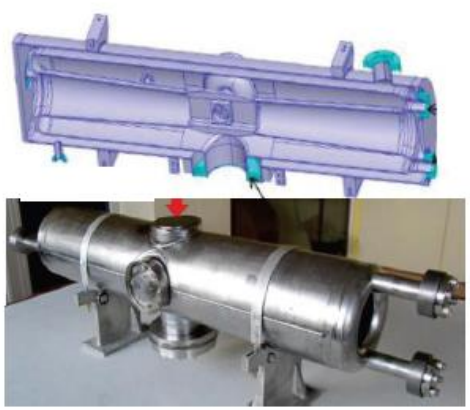
Cavity examples – Half wave resonators

IMP β=0.10, f=162.5MHz

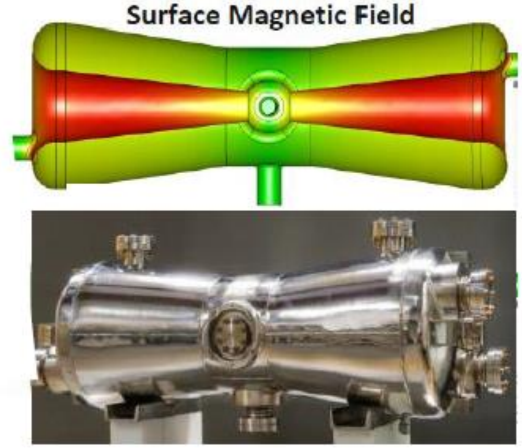


FRIB β=0.29, 0.53 f=322MHz

FRIB β=0.29, 0.53 f=322MHz



IFMIF β=0.11, f=175MHz



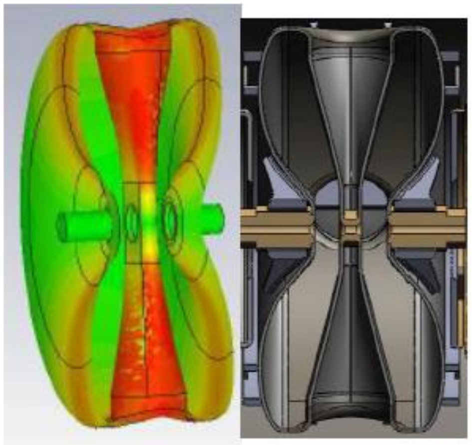
ANL β=0.112, f=162.5MHz

Typical range

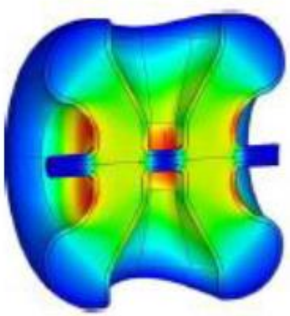
β~0.10->0.5

f=140MHz->325MHz

Cavity examples – Single Spoke Resonators

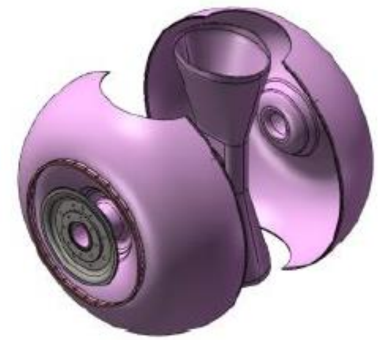


IHEP β=0.12, f=325MHz



FNAL 325 MHz, $\beta_0 = 0.47$

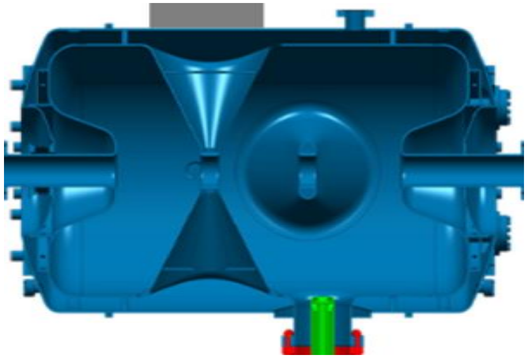
FNAL β=0.215, f=325MHz



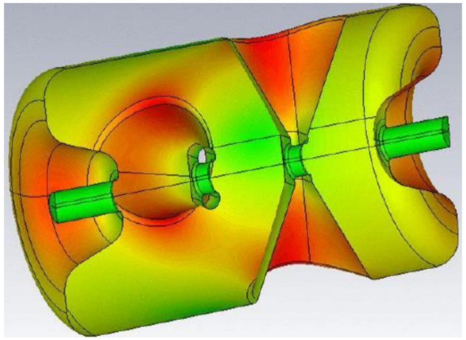
TRIUMF/RISP β=0.3, f=325MHz

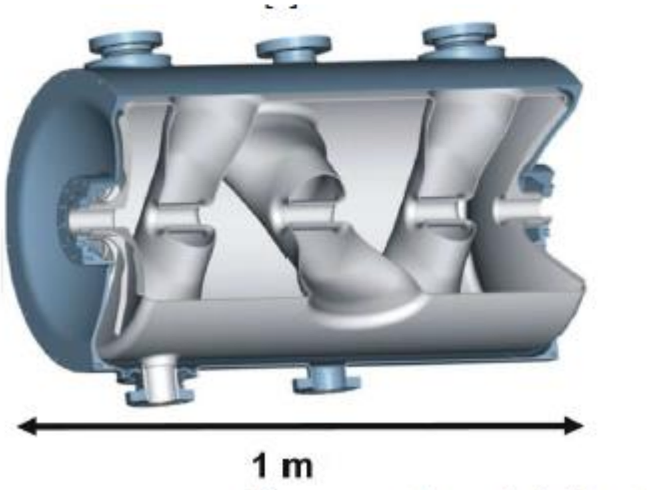
Typical range
β~0.15->0.7
f=325-700MHz

Cavity examples – Multi-cell Spoke Resonators

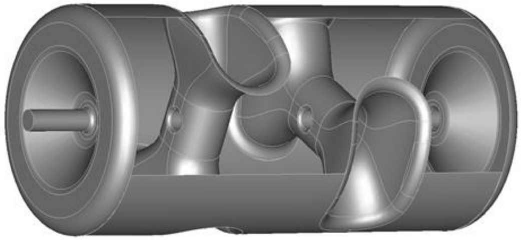


ESS/IPN β=0.50, f=352MHz



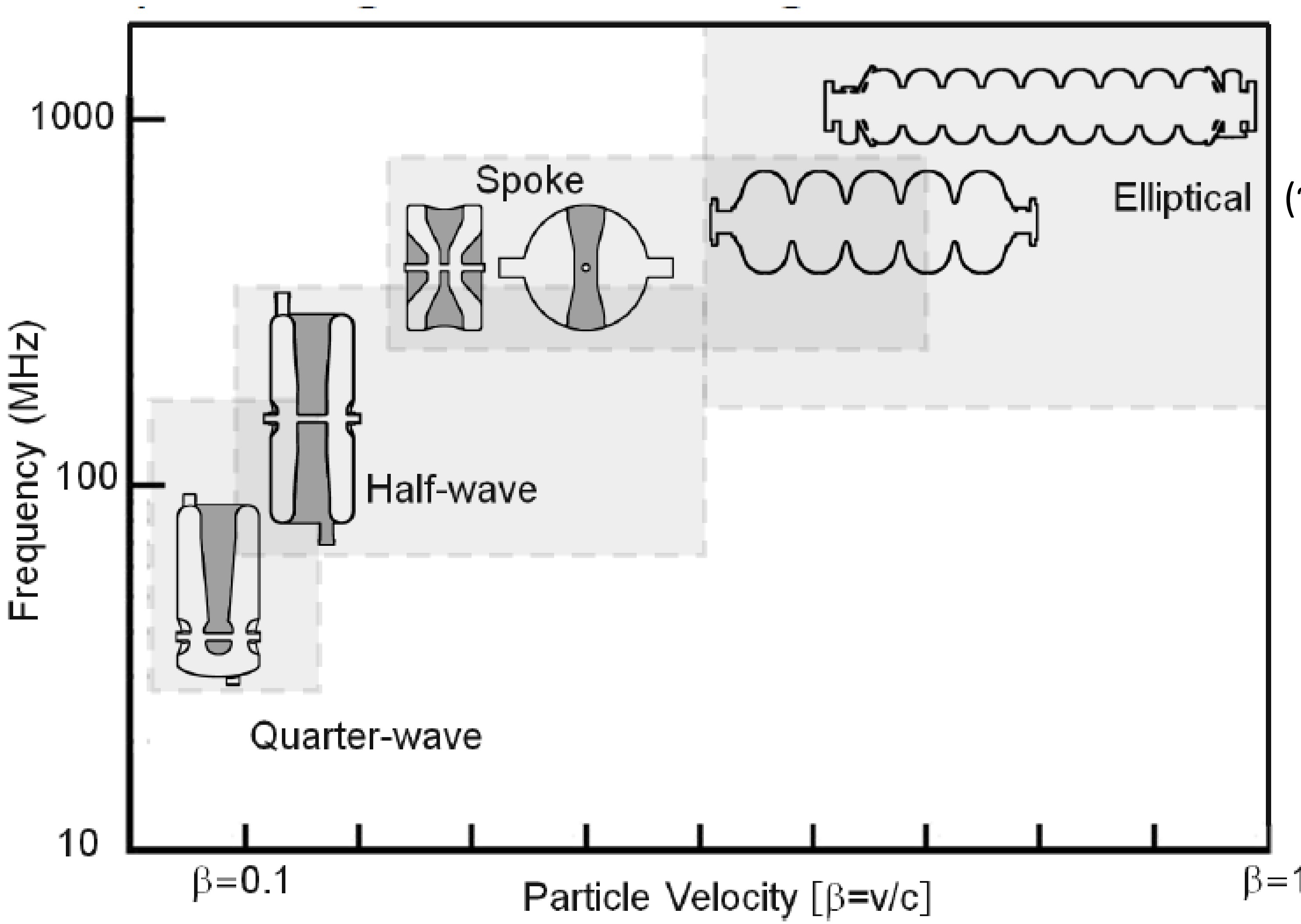


ANL β=0.63, f=345MHz



500 MHz, $\beta_0 = 1$
Double-Spoke Cavity

Cavity type / velocity / frequency chart



(~pill-box)

Other cavity types

Deflecting mode cavities

- There has been a resurgence in cavities designed to provide a phase dependent transverse deflection to the beam using a TE-like (H-mode) dipole electric field
- Applications include crab cavities and rf kickers
- Deflection from both electric and magnetic fields
- Frequency governed by transverse dimension (~ λ/2)
- The variants have a high deflecting shunt impedance in a relatively compact geometry

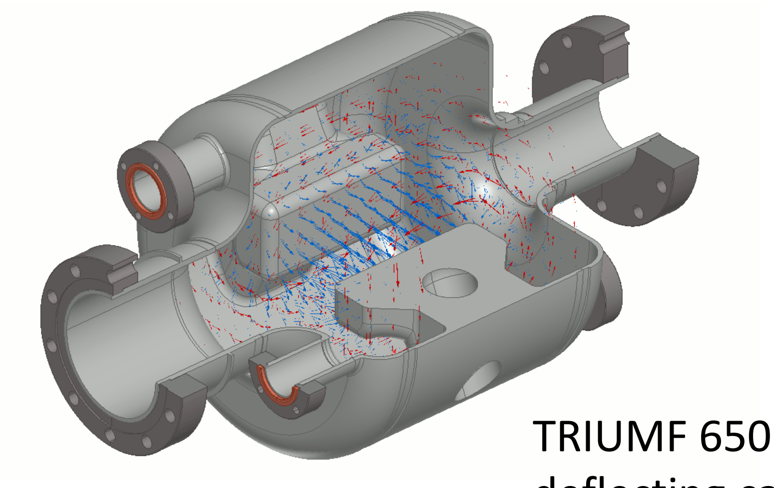
$$V_\perp = \int_{-\infty}^{\infty} \left[E_x(0,z)\cos\left(\frac{\omega}{\beta c} z\right) + \beta c B_y \sin\left(\frac{\omega}{\beta c} z\right)\right] \cdot dz$$

$$R_\perp = \frac{V_\perp^{\,2}}{P_c} = \frac{V_\perp^{\,2} Q_0}{\omega U} \quad \text{so} \quad \frac{R_\perp}{Q_0} = \frac{V_\perp^{\,2}}{\omega U}$$

Deflection angle is given by $\frac{\Delta p_x}{p_z}$ where

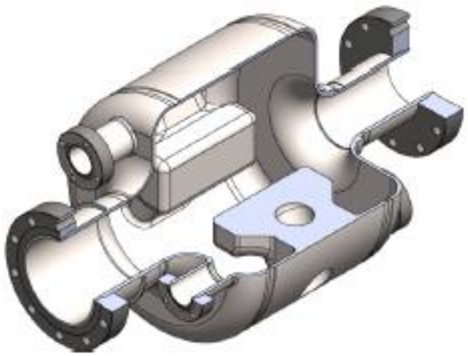
$$\Delta p_x = \int F_x dt = \int F_x \frac{dz}{\beta c} = \frac{1}{\beta c}\int F_x dz = \frac{V_\perp}{\beta c}$$

Therefore $\frac{\Delta p_x}{p_z} = \frac{V_\perp}{\beta c}\frac{1}{\beta \gamma m_0 c} = \frac{V_\perp}{\beta^2 E}$ where E is total energy



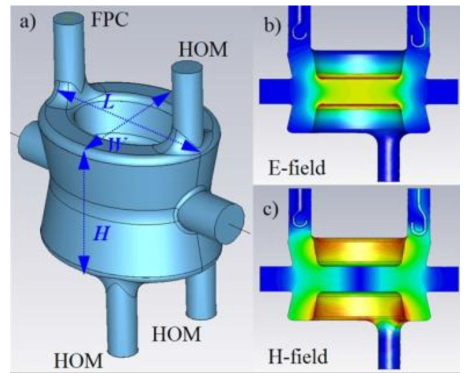
TRIUMF 650MHz deflecting cavity

Deflecting mode cavities

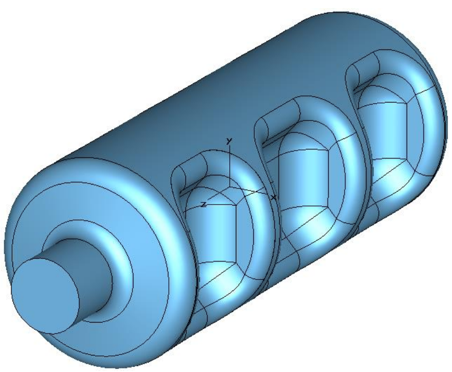


TRIUMF 650MHz

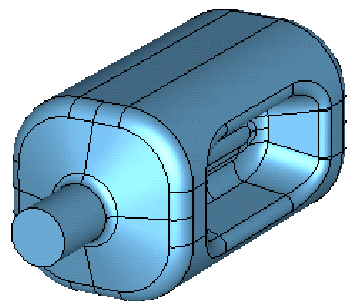
double quarter wave (DQW) – 400MHz – BNL/CERN

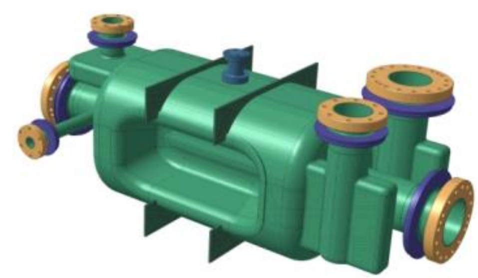


RFD – multi-cell – 953MHz – ODU



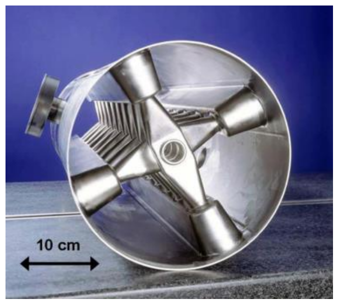
RF Dipole (RFD) – 400MHz – ODU/CERN

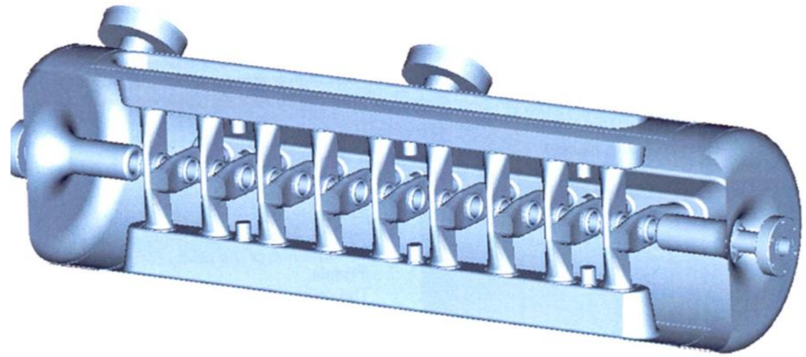




Cross bar TE mode - CH

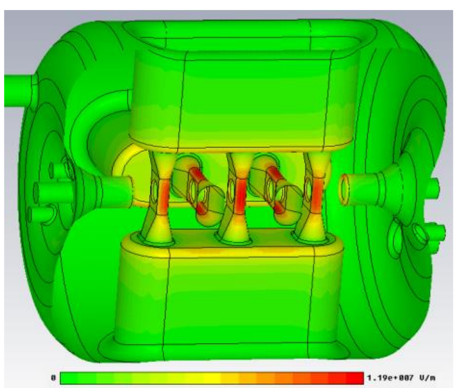
- Another multi-gap variant is the cross-bar H-mode cavity
- Spokes stem from four ridges with opposite ridges having the same voltage and adjacent ridges in anti-phase
- Designed to give a high accelerating voltage in a fixed velocity regime



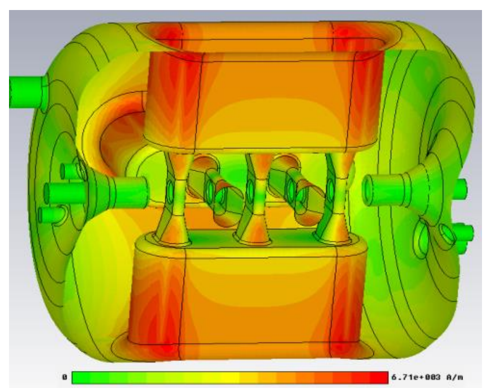


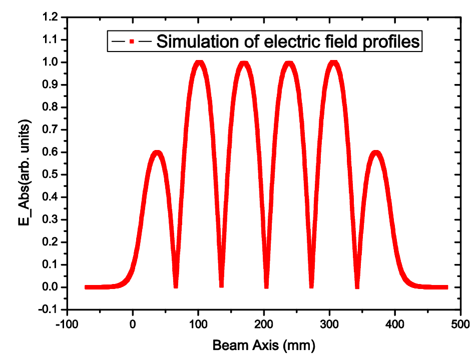
360 MHz, β_0~0.1 19 gap CH resonator

E- field



H- field

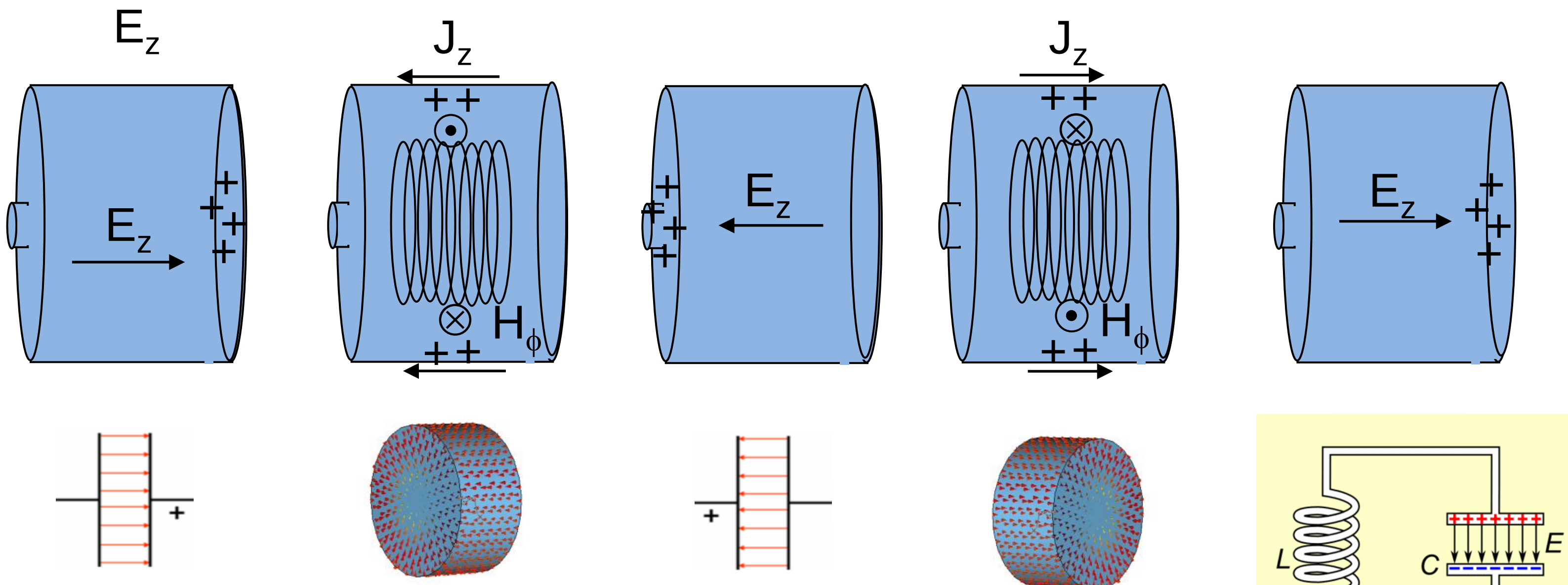




CH 6 cell fabricaated by IMP – 162.5MHz β=0.067

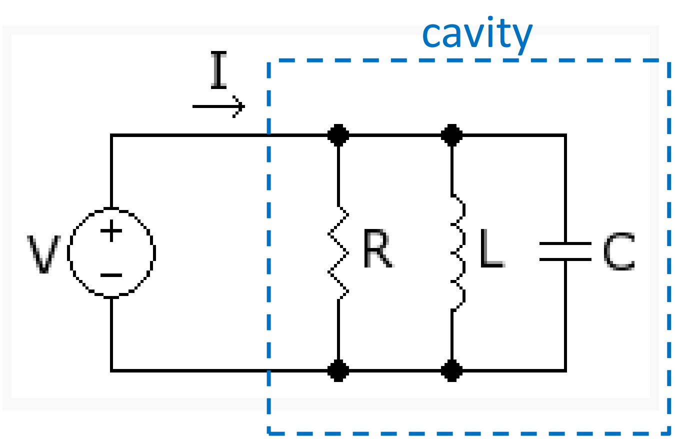
Cavity Circuit Representation

Cavity circuit representation vs pill-box



Capacitance

Inductance
Resistance

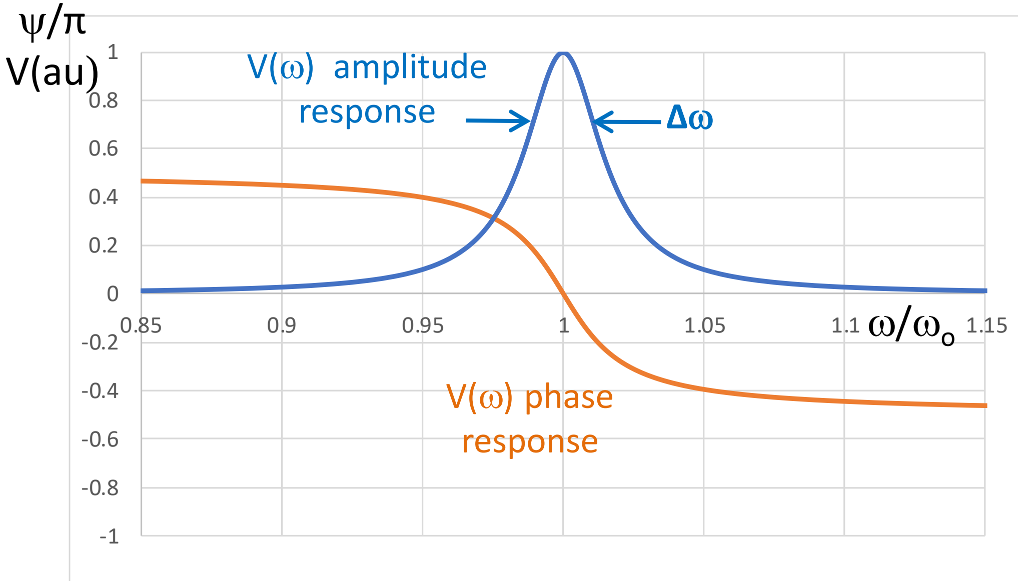


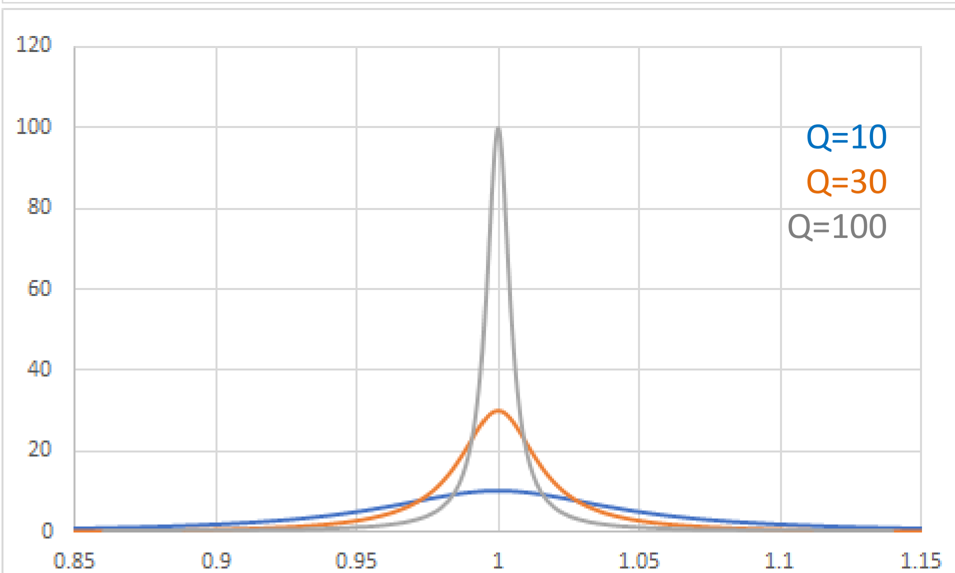
Note the comparison to the LCR parallel circuit.

The parallel resonant circuit driven be a current generator is a good model for describing a single mode of an accelerating cavity.

Fields build resonantly driven by an rf source with damping by the electrical resistance in the cavity walls.

LCR Circuit – Resonance Curve





- By convention the bandwidth of an oscillator is defined as the frequency difference between the ½ power points or where $V(\omega)=0.7V(\omega_0)$, we define Q as the quality factor of the resonance

$$\Delta\omega = \frac{\omega_0}{Q}, \quad Q = \frac{\omega_0}{\Delta\omega}$$

- The stored energy and cavity voltage of a LCR circuit is given by:

$V_{rms}^2 = V_0^2/2$

$$U = \frac{CV_0^2}{2} \quad \text{and} \quad P = \frac{V_0^2}{2R},$$

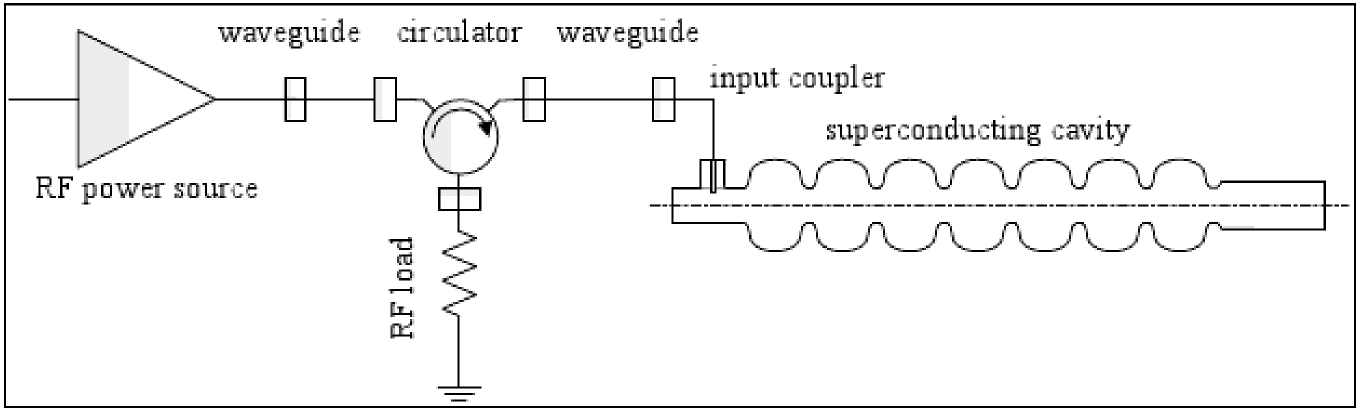
$$\text{so} \quad Q = \frac{\omega_0 U}{P} = \omega_0 RC = \frac{R}{\omega_0 L}$$

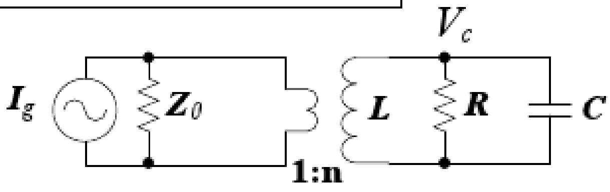
$$\frac{R}{Q} = \omega_0 L = \sqrt{\frac{L}{C}}$$

Figure of Merit
Next Lecture

Connecting to a power source

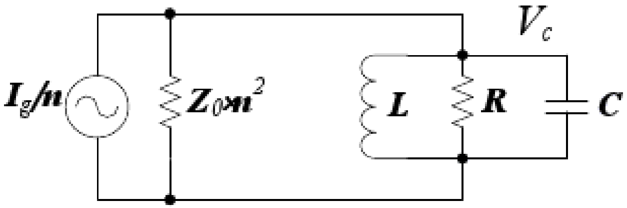
A cavity is connected via a transmission line to an rf power source – the power is fed to the cavity via an input coupler – reflected power is returned back toward the source and is usually absorbed in a circulator.





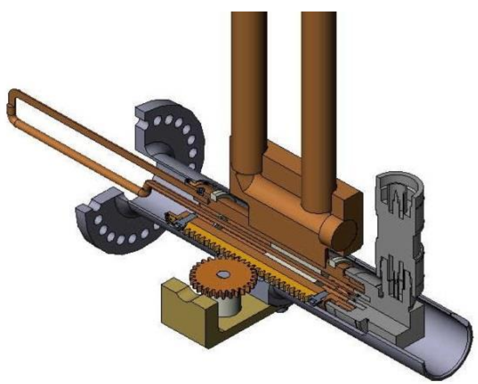
The combined system can be represented by an AC circuit model with the coupling modeled as a 1:n transformer

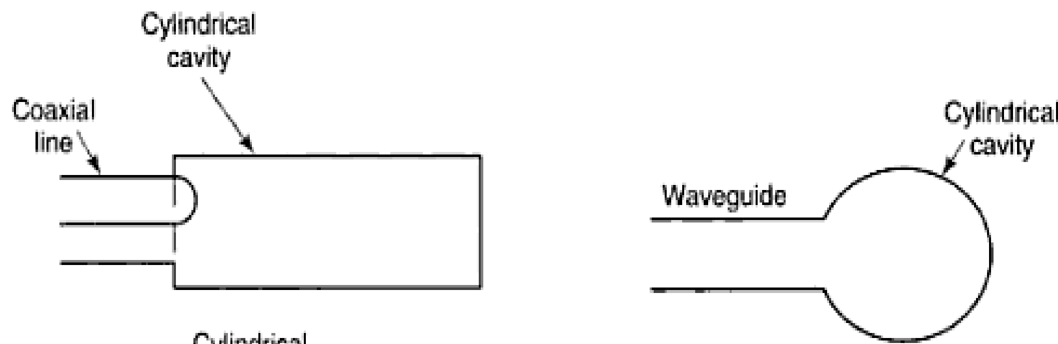
Coupler acts like an external impedance $Z_0\,n^2$

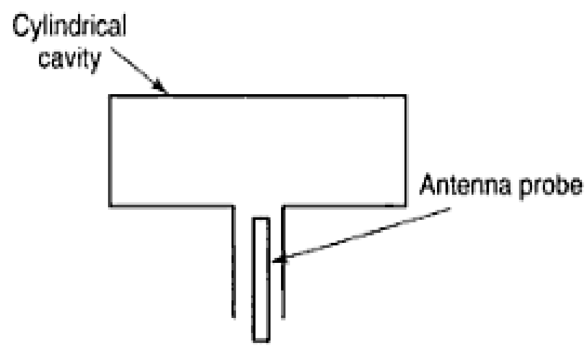


Coupling Power into the Resonator

- Power is fed to the cavity through inductive or capacitive couplers or couple directly with a waveguide
- The coupler provides a small excitation to the cavity that builds field on each cycle when the cavity is near resonance – similar to any driven resonant system







Methods of coupling to cavities

SRF Cavity-I (summary)

- RF accelerating systems typically consist of high Q resonant cavities that are excited by radio frequency electromagnetic fields
  - Consist of evacuated space enclosed by conducting walls – can sustain an infinite number of resonant electromagnetic modes
  - Shape is selected so that a particular mode can efficiently transfer its energy to a charged particle
- Time-dependent electromagnetic fields interact with the charged particles to transfer energy – only E fields can accelerate
- Different cavity types (elliptical, QWR, HWR, SSR) are available with the choice dependent on the particle velocity and available rf frequencies
- The structures (cavities) are tuned to resonate at a given frequency and are driven by external rf amplifiers; the resonance is damped by the resistance in the walls